GX1336

OLLSCOIL NA hÉIREANN
National University of Ireland, Galway

SEMESTER II
SUMMER EXAMINATIONS 1999/2000

Second University Examination in Information Technology

LOGICAL FOUNDATIONS OF COMPUTING (CT214)

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Time Allowed : **Three Hours**

Candidates should attempt four questions, two from each section.
Please use separate answer books for each section.
All questions carry equal marks

SECTION A

A1. (i) Show how to write down the disjunctive normal form (DNF) of a gate $f: Z_2^n \to Z_2$, and explain briefly why the DNF takes the value 1 precisely when f does.

(ii) Find the logic table and the DNF of the boolean expression

$$(x_1 \wedge \overline{x_2}) \vee (x_2 \wedge x_3)$$

(iii) There are three tests used in diagnosing a certain disease. A patient who reacts positively to the first test and negatively to at least one of the other two is judged to have the disease. Construct a basic machine which models this diagnostic procedure, and explain the notation you use.

A2.(i) Show that the argument P, Q $\therefore$ C is *valid* if and only if the compound propositions P,Q,~C are *inconsistent*. Define carefully the terms in italics.

(ii) A mathematician argues as follows:
"If the graph is bipartite and has a Hamiltonian cycle then the two parts have the same size. The graph is bipartite buts the parts have different sizes. So the graph is not Hamiltonian."
Using b, h and s for "the graph is bipartite", "the graph is Hamiltonian" and "the parts are the same size", construct the argument of which the above reasoning is an instance. Test for validity the argument you have constructed.

(iii) In a certain firm the following pension rule applies:
"to get a pension an employee must have at least 20 years service".
Four former employees tell me they had (respectively):
(a) 30 years service (b) a pension (c) no pension (d) 10 years service
Which must I question further to see if the rule was applied correctly? Justify your answer.

A3.(i) Describe the <u>tableau</u> method for testing a collection of well-formed formulae (compound propositions) for consistency/inconsistency.

(ii) Use the <u>tableau</u> method to show that the following collection of well-formed formulae (WFFs) is consistent:

$$\sim(p \to q), \quad q \vee r, \quad p \to r$$

Read off, from the tableau, an assignment of values for p, q and r which makes all three WFFs take the value 1.

(iii) Use the <u>resolution</u> method to show that the following argument is valid:

$$(\sim p) \vee (q \wedge r), \quad q \to (p \wedge (\sim r)) \ \therefore\ \sim p$$

SECTION B

B1. (i) Using laws of the propositional calculus, prove the following:

a) $p \vee (p \wedge q) = p \wedge (p \vee q)$

b) $(p \vee q) \wedge \sim(p \wedge q) = (p \wedge \sim q) \vee (\sim p \wedge q)$

c) $p \Rightarrow (q \Rightarrow p)$

d) $(p \Rightarrow q) \Rightarrow p) = p$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

(ii) Prove that a possible alternative definition for implication is $(p \Rightarrow q) =_{def} (p \equiv (p \wedge q))$.

(iii) Use the propositional calculus to show that the following argument is valid:
If the programming team is happy, the specification is clear. The specification is not clear. Therefore the programming team is not happy.

B2. (i) Use the following definitions:

$$(p \equiv q) =_{def} (p \wedge q) \vee (\sim p \wedge \sim q)$$
$$(p \not\equiv q) =_{def} (p \wedge \sim q) \vee (\sim p \wedge q)$$

to prove:

a) $p \equiv p$

b) $\sim(p \not\equiv p)$

c) $(p \not\equiv q) = \sim(p \equiv q)$

(ii) Argue that the proposition

$$p_1 \not\equiv p_2 \not\equiv \ldots \not\equiv p_n$$

is true (1) if and only if an odd number of the $p_1, p_2, \ldots p_n$ are true.

(iii) On a certain island of knights and knaves, knights always tell the truth and knaves always lie. A says, "If I am a knight then B is a knave". What are A and B?

B3. (i) Represent the following statements using predicate calculus:

a) If Sarah is Paul's manager then she either has more years experience than Paul or is an expert in OOD and C++.

b) Every employee who is an expert in C++ is a programmer.

c) The leader of any project team has at least 3 years' experience.

Assume that *E* is the universe of employees, *PT* is the universe of project teams, *J* is the universe of job titles, and *S* is the universe of areas of expertise. You may use the following atomic predicates:

EmployedAs(j,p) indicates whether employee *p* has job title *j*.
Leader(g,e) indicates whether employee *e* leads project group *g*.
ExpertIn(s,e) indicates whether employee *e* is has expertise *s*.
Manages(e1,e2) indicates whether employee *e1* is managed by *e2*.

In addition, the function *Experience(e)* will give the number of years experience that employee e has.

(ii) Justify the generalisation of de Morgan's law for the universal quantifier:

$\sim(\forall x{:}U \bullet P(x)) = \exists x{:}U \bullet \sim P(x)$

Use this law to prove:

$\sim(\forall x{:}U \bullet R(x) \Rightarrow P(x)) = \exists x{:}U \bullet R(x) \wedge \sim P(x)$

(iii) If no students turn up for an examination, is it true to say that (a) all students who took the examination failed? (b) all students who took the examination passed? Justify your answer.