

OLLSCOIL NA hÉIREANN, GAILLIMH  
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2000

**MATHEMATICAL ECONOMICS – EC 218**

2<sup>nd</sup> year Arts Students

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Time allowed: **TWO** hours

Answer any **THREE** questions

1. The ABC Company is going to introduce new products: a widget, a hummer, or a nimnot. The market conditions (favourable, stable or unfavourable) will determine the profit or loss the company realises, as shown in the following pay-off table:

Product	Market Conditions		
	Favourable (.2)	Stable (.7)	Unfavourable (.1)
Widget	240,000	140,000	-60,000
Hummer	120,000	80,000	40,000
Nimnot	70,000	60,000	60,000

Where the numbers in brackets are the probabilities of each market state.

- (a) Compute the expected value for each decision and select the best one.
- (b) The ABC company wants to hire a market research firm to do a survey to determine future market conditions. The results of the survey will indicate either positive or negative market conditions. There is a .60 probability of a positive report given favourable conditions, a .30 probability of a positive report given stable conditions, and a .10 probability of a

positive report given unfavourable conditions. There is a .90 probability of a negative report given unfavourable conditions, a .70 probability given stable conditions, and a .40 probability given favourable conditions. Using decision tree analysis and posterior probability tables, determine the decision strategy the company should follow, the expected value of the strategy, and the maximum amount the company should pay the market research firm for the survey results.

2. The Siberian Oil Company is considering a bid for a shale oil development contract to be awarded by the federal government. The company estimates that it has a 60% chance of winning the contract with this bid. If the firm wins the contract, it can choose one of three methods for getting the oil from the shale. It can develop a new method for oil extraction, use an existing (inefficient) process, or subcontract the processing to a number of smaller companies once the shale has been excavated. The results from these alternatives are given as follows:

Develop new process:

Outcomes	Probability	Profit (millions)
Great Success	.3	600
Moderate Success	.6	300
Failure	.1	-100

Use present process

Outcomes	Probability	Profit (millions)
Great Success	.5	300
Moderate Success	.3	200
Failure	.2	-40

Subcontract:

Outcomes	Probability	Profit (millions)
Moderate Success	1.0	220

The cost of preparing the contract proposal is \$2 millions. If the company does not make a bid, it will invest in an alternative venture with a guaranteed profit of \$50 millions. Construct a sequential decision tree for this decision situation and determine whether the company should make a bid.

3. In a duopoly with homogenous products, the rival firms face the inverse market demand  $p = 260 - Q$ , and both have identical marginal costs of \$20, where

$P$  = price of output

$Q = (q_1 + q_2)$

$q_1, q_2$  = output of firms 1 and 2 respectively.

- (a) Solve for the Cournot-Nash equilibrium in this model, stating clearly your assumptions, and calculate the equilibrium price, quantities and profit levels for the two firms.
- (b) Solve for the Stackelberg equilibrium in this model, assuming that firms move sequentially and that firm 1 moves first.
- (c) Explain the distinction between the two types of game specified in (a) and (b) above.

4. The law firm of Cline and Pine employs three types of lawyers: junior lawyers, senior lawyers and partners. During a given year, there is a .15 probability that a junior lawyer will be promoted and become senior lawyer and a .05 probability that he or she will leave the firm. Also there is a .20 probability that a senior lawyer will be promoted to partner and a .10 probability that he or she will leave the firm. There is also a .05 probability that a partner will leave the firm.

The firm never demotes a lawyer.

- (a) What is the probability that a newly hired junior lawyer will leave the firm before becoming a partner?
- (b) On average, how long does a newly hired junior lawyer stay with the firm?

- (5) In a quantity setting duopoly with homogenous product, the inverse market demand function is given by:

$$P = 320 - Q, \text{ where}$$

$$Q = (q_1 + q_2)$$

$q_1, q_2$  = output of firms 1 and 2 respectively.

The constant marginal cost for each firm is \$80.

- (a) Find the collusive outcome where the firms act as a joint monopolist. How does it differ from the Cournot-Nash equilibrium?
- (b) Can a subgame-perfect Nash equilibrium that supports monopoly payoffs be achieved for this Cournot market game infinitely repeated? Analyse the role of the interest rate or discount factor in achieving the equilibrium.
- (c) Comment on the likelihood and sustainability of collusion when the number of firms in the industry grows significantly.