

OLLSCOIL NA hÉIREANN, GAILLIMH  
NATIONAL UNIVERSITY OF IRELAND, GALWAY

Summer Examinations 2000

**Techniques of Analysis in the Social Sciences (EC 222)**

2<sup>nd</sup> B.A. (Economic and Social Studies)

Prof. Michael Cuddy  
Prof. Jeffrey Round  
Ms. Breda Lally

**Time Allowed: TWO HOURS**

**Marks: 300**

**Instructions:** This exam consists of three sections. You must answer questions from all three sections. Please read carefully the instructions for each section.

- **Section A (45 Marks):** Answer Question 1 OR Question 2
- **Section B (105 Marks):** Answer **THREE** questions (worth 35 marks each)
- **Section C (150 Marks):** Answer **THREE** questions. You **MUST** answer Question 1 (worth 30 marks) **AND EITHER** Question 2 **OR** Question 3 (worth 60 marks each). The third question must be chosen from the remaining questions.

**Section A**

1. An important part of the customer service responsibilities of Eircom relate to the speed with which troubles in residential service can be repaired. Suppose that past data indicate that the likelihood is 0.70 that troubles in residential service can be repaired on the same day. For the first five troubles reported on a given day, what is the probability that
  - (a) all five will be repaired on the same day?
  - (b) at least three will be repaired on the same day?
  
2. The Institute of Chartered Accountants wants to study the characteristics of its' 1999 graduates. It decides to focus on two variables: the amount of money earned by graduates, and where graduates received their training. The results from a sample of 80 graduates are as follows:
  - Amount of money earned:  $\bar{x} = £28,500$ ,  $s = £238$
  - 40 graduates trained in Dublin, 25 trained in Cork and 15 trained in Galway
  - (a) Set up a 95% confidence interval estimate of the population average amount earned by accountants.
  - (b) Set up a 95% confidence interval estimate of the proportion of accountants who trained in Galway.

## Section B

1. The quality control manager at a light bulb factory needs to estimate the average life of a large shipment of light bulbs. The population standard deviation ( $\sigma$ ) is known to be 90 hours. A random sample of 60 light bulbs indicated a sample average life ( $\bar{X}$ ) of 320 hours. Set up a 99% confidence interval estimate of the true average life of light bulbs in this shipment.
2. A carpet manufacturer is studying differences between two of its major outlet stores. The company is particularly interested in the time it takes customers to receive carpeting that was ordered from the plant. Data concerning a sample of delivery times from store 1 and store 2 for the most popular type of carpet are summarized as follows:

$$n_1 = 41 \quad S_1^2 = 5.76 \quad n_2 = 31 \quad S_2^2 = 9.61$$

At the 0.05 level of significance, is there evidence of a difference in the variances of the shipping time between the two outlets?

3. The manager of the credit department for an oil company would like to determine the average monthly balance of credit card holders. A random sample of 100 accounts indicates a sample average balance of £83.40 with a sample standard deviation of £23.65. Using the 0.05 level of significance, is there evidence that the average monthly balance is different from £75?
4. Toby's Trucking Company determined that on an annual basis, the distance traveled per truck is normally distributed with a mean ( $\mu$ ) of 50,000 miles and a standard deviation ( $\sigma$ ) of 12,000 miles. If a sample of 16 trucks is selected what is the probability that the average distance traveled is between 44,000 and 48,000 miles per year?
5. A claim is made that 70% of graduates in Ireland find a job within 6 months of graduation. To determine the validity of this claim, the Department of Education selects a sample of 200 graduates and finds that 155 have found a job within 6 months. Is there evidence at the 0.05 level of significance that the claim is not valid?

## Section C

1. Consider the following basic Keynesian model of the macro economy:

$$Y = C + I_0 + G_0$$

$$C = a + bY$$

where  $Y$  and  $C$  are endogenous,  $I_0$  and  $G_0$  are exogenous constants,  $a > 0$ , and  $0 < b < 1$ .

Use Cramer's Rule to solve for  $Y$  and  $C$ .

2. (a) Show that profit is maximized/minimized when  $MR = MC$ . How would you determine whether profit is a maximum and not a minimum when  $MR = MC$ ?
- (b) A shop which sells T-shirts has a demand function and a total cost function given by the equations

$$P = 248 - 10Q \quad \text{and} \quad TC = 120 + 8Q$$

- (i) Write down the equations for TR, profit, MC and MR
- (ii) Calculate the number of T-shirts that must be sold to maximize profit. Hence calculate maximum profit.
3. The quantity of sandwiches,  $Q$ , made in a small coffee shop is given by the equation

$$Q = -2L^3 + 12L^2$$

where  $L$  is the number of labour-hours hired.

- (a) Write down the equations for the marginal product of labour ( $MP_L$ ) and average product of labour (APL).
- (b) Calculate the units of labour at which the  $MP_L$  and APL are maximized.
- (c) Confirm algebraically that  $MP_L$  and APL are equal when APL is at a maximum.
4. Using matrix inversion, solve the following system of equations for  $x$ ,  $y$  and  $z$ .

$$\begin{aligned} x &= 12 - y \\ 5y + 2z &= 20 - 2x \\ 6x + 3y + 6z &= 0 \end{aligned}$$

5. Assume a production function of the form:

$$Q = 10L^{0.75} K^{0.25}$$

- (i) Find the partial derivatives of labour and capital.
- (ii) Find all the second-order partial derivatives.
- (iii) Find the total differential.
- (iv) For answers (i) and (ii) explain the meaning of each.

6. A perfectly competitive firm produces two goods, X and Y, which are sold at £54 and £52 per unit respectively. The firm has a total cost function given by

$$TC = 3x^2 + 3xy + 2y^2 - 100$$

Find the quantities of each good that must be produced and sold in order to maximize profits.