GX374

National University of Ireland, Galway

Ollscoil na hÉireann, Gaillimh

SUMMER EXAMINATIONS 2000

M.A.(Applied Economics)/M.Econ.Sc.

ECONOMETRICS EC506

Dr. Jeff Round

Professor M. P. Cuddy

Dr. Jennifer Stewart

INSTRUCTIONS

Time Allowed: 3 Hours

There are two parts to this exam for a total of 100 marks. Answer all questions in the booklet provided.

Part A is compulsory and is worth a total of 25 marks.

In Part B, choose 3 out of the 6 questions to answer (25 marks per question for a total of 75 marks). Only the first three questions answered will be marked. Marks will only be given if work is shown.

This examination is an open-book exam. Students are allowed to bring in any material they wish. Students are NOT allowed to share material during the exam. Calculators are allowed.

Part A. (25 marks)

The output below is from a OLS regression run to estimate $y_t = X_t'\beta + e_t$ where y_t is welfare participation rates and X_t includes:

1. dummy variables for the 9 regions,
2. benefit level which varies over time,
3. a measure of the unemployment insurance generosity which varies between regions and over time,
4. the unemployment rate which which varies between regions and over time,
5. minimum wage earning which varies over time, and
6. dummy variables for the quarter.

The data covers 82 quarters for each region so the total sample size is 738.

Valid cases:	738	Dependent variable:	Y
Missing cases:	0	Deletion method:	None
Total SS:	1366.757	Degrees of freedom:	722
R-squared:	0.891	Rbar-squared:	0.889
Residual SS:	149.173	Std error of est:	0.455
F(16,722):	368.321	Probability of F:	0.000
Durbin-Watson:	0.289		

Variable	Estimate	Standard Error	t-value	Prob $> \|t\|$	Standardized Estimate	Cor with Dep Var
Region 1	0.170274	0.016475	10.335022	0.000	0.881089	0.889136
Region 2	-0.298692	0.069951	-4.270009	0.000	-0.262275	0.747230
Region 3	0.161769	0.010029	16.130912	0.000	0.603140	0.884367
Region 4	1.104700	0.043005	25.687954	0.000	3.282742	0.879711
Region 5	-0.138370	0.049451	-2.798148	0.005	-0.026942	0.430901
Region 6	-0.151061	0.049270	-3.065972	0.002	-0.030848	0.416620
Region 7	-0.024923	0.049024	-0.508394	0.611	-0.004853	0.388284
Region 8	-8.551391	0.438804	-19.487959	0.000	-1.123807	0.329995
Region 9	-9.404474	0.459593	-20.462630	0.000	-1.235918	0.266183
Benefit Level	-9.380717	0.449273	-20.879763	0.000	-1.232796	0.233080
UI System	-8.984176	0.449921	-19.968335	0.000	-1.180683	0.306939
Unemployment Rate	-8.549210	0.443132	-19.292676	0.000	-1.123521	0.330434
Minimum Wage	-9.177243	0.458373	-20.021344	0.000	-1.206056	0.280662
Quarter 1	-9.240487	0.439139	-21.042271	0.000	-1.214367	0.247797
Quarter 2	-9.017069	0.452993	-19.905523	0.000	-1.185006	0.343184
Quarter 3	-9.714111	0.460030	-21.116258	0.000	-1.276610	0.193370

Given the nature of the data what tests should a researcher conduct next? Describe the tests that you would conduct. Explain why you would conduct these tests. How could the results from the tests change your estimation technique?

Part B. (Choose 3 questions from the following 6 questions. Each question is worth 25 marks)

Question 1

Confirm that the normal equation $(X'X)b = X'y$ yields the same results as in the 2-variable case.

Question 2

The OLS equation $y = b_1x_1 + b_2x_2 + e$ is fitted by OLS based on 10 observations and it is found that $b_1 = 5$, its t-statistic is 2.5, and the residual sum of squares is 40.
When x_1 is regressed on x_2 the uncentered R^2 is 0.5. (That is, the residual sum of squares from that regression is 0.5 times $x_1'x_1$.)
The smaller OLS equation $y = b_1^*x_1 + e^*$ gives an estimate of $b_1^* = 7$ and a residual sum of squares of 90. Compute the t-statistic for b_1^* that would be given by this smaller regression.

Question 3

Consider the model $y = X\beta + e$, where $E(e) = 0$ and $E(ee') = \sigma^2 I$. Using the data on y and X, it was found that:

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{pmatrix} 0.5 & -2 & -0.1 & 0 \\ -2 & 10 & 1 & 0 \\ -0.1 & 1 & 0.5 & 0.1 \\ 0 & 0 & 0.1 & 1 \end{pmatrix}, \mathbf{X}'\mathbf{y} = \begin{pmatrix} 20 \\ 0.5 \\ 2 \\ -2 \end{pmatrix},$$

$$\mathbf{y}'\mathbf{y} = 417.7, \mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} = \begin{pmatrix} 8.8 \\ -33 \\ -0.7 \\ -1.8 \end{pmatrix}$$

1. The estimated variance of the error term is $s^2 = 16$. Describe how this could be have been computed from the above information and knowing that there are 20 observations. (You do not have to make this computation.)(5 marks)

2. Obtain t-statistics for the following null hypotheses:

   (a) $\beta_2 = 0$

   (b) $\beta_3 = \beta_4$

   (5 marks)

3. Obtain F-statistics for the following null hypotheses:

   (a) $\beta_3 = 0$

   (b) $\beta_2 = 0 \text{and} \beta_3 = \beta_4$

   (5 marks)

4. Give a prediction of y and its 95% confidence interval when the vector $X = [1, 0, 1, 0]$. (5 marks)

5. Let e_i denote the OLS residual of observation i. Suppose that

$$\frac{\Sigma_2^{20} e_i e_{i-1}}{\Sigma_1^{20} e_i^2} = 0.4$$

   Give the approximate value of the Durbin-Watson statistic. (5 marks)

Question 4

A researcher estimates the linear relationship, $y = \alpha + \beta x + e$, and the associated standard errors by applying OLS to the data:

X	2	3	1	5	9
Y	4	7	3	9	17

She is subsequently informed that the variance matrix for the disturbances underlying the data is

$$var(e) = \sigma^2 \cdot \text{diag}[0.10, 0.05, 0.20, 0.30, 0.15]$$

Use this information to calculate the correct standard errors for the OLS estimates and compare with those obtained from the conventional formula.

Question 5

Suppose the OLS model applies to $y = \beta x + e$ except that e is first-order autocorrelated with autocorrelation coefficient $\rho = 0.5$ and variance 9. You have two observations on x and y; the first observations are $x = 1$ and $y = 4$, and the second observations are $x = 2$ and $y = 10$.

1. What is the OLS estimate of β?(8 marks)

2. What is the GLS estimate of β?(8 marks)

3. What are the variances of these estimates?(9 marks)

Question 6

1. Write down models which statisfy the following conditions (i.e., specify the coefficients numerically); verify that the conditions are satisfied:

   (a) an ARMA(1,0) model that is stationary;

   (b) an ARMA(2,0) model that is stationary;

   (c) an ARMA(2,0) model that is nonstationary and explosive;

   (d) an ARMA(2,0) model that is nonstationary and nonexplosive;

   (e) an ARMA(2,0) model that is nonstationary, but can be made stationary by a differencing transformation. (10 marks)

2. Prove that an MA(q) model with q finite is always stationary.(8 marks)

3. If q is infinite in the MA(q) model, what is the requirement for stationarity? (7 marks)