GX519

OLLSCOIL na hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SECOND SEMESTER EXAMINATIONS, 2000

B.E. DEGREE EXAMINATION

COMPUTATIONAL METHODS IN CIVIL ENGINEERING

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Time allowed: *Three* hours
Answer *five* questions

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1. In a consolidation test a soil specimen is placed between layers of porous stone and subjected to an applied stress of constant value of p_0. As drainage occurs only in the vertical direction, the change in pore water pressure during the test can be described by the one-dimensional consolidation equation.

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

where c_v is the coefficient of consolidation and u is the pore water pressure.
Derive an explicit finite difference approximation for this problem and use it to determine the pore water pressure distribution at 10 equal intervals through the depth of the soil sample at 5 time steps Δt from the start of the test. Comment on the selection of a suitable time step for this problem.

2. The potential energy of a beam of flexural rigidity EI and length L when subject to a transverse loading q(x) per unit length may be written as

$$\Pi = \frac{1}{2}\int_L EI\left(\frac{d^2w}{dx^2}\right)^2 dx - \int_L qw\,dx$$

where w(x) is the lateral deflection.
In a finite element discretisation of the beam, two-noded beam elements based on the first order Hermite polynomials and having nodal degrees of freedom w and θ are used.
Derive the stiffness equation for this element. Evaluate two terms in the stiffness matrix and one term in the nodal load vector when the element is subject to a uniformly distributed load q_0.

3. The temperature distribution, $T(x)$, due to conduction of heat through a slab of thickness H having a thermal conductivity $k(x)$ is governed by the equilibrium equation

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+Q=0 \qquad 0\le x\le H$$

subject to the boundary conditions

$$k\frac{dT}{dx}=\bar{q} \qquad \text{at } x=0$$

$$T=T_B \qquad \text{at } x=H$$

where Q is the rate of internal heat generation.

Use Galerkin's method to derive the finite element stiffness equation for this problem.

4. Show how area co-ordinates can be used as shape functions for plane triangular finite elements. Using area co-ordinates, derive the shape functions for a quadratic triangular element for C^0 problems.

Silvester's formula for an nth order element may be written as

$$N_{\alpha\beta\gamma}(L_1, L_2, L_3) = N_\alpha(L_1)N_\beta(L_2)N_\gamma(L_3)$$

where L_1, L_2 and L_3 are the area co-ordinates of the element and

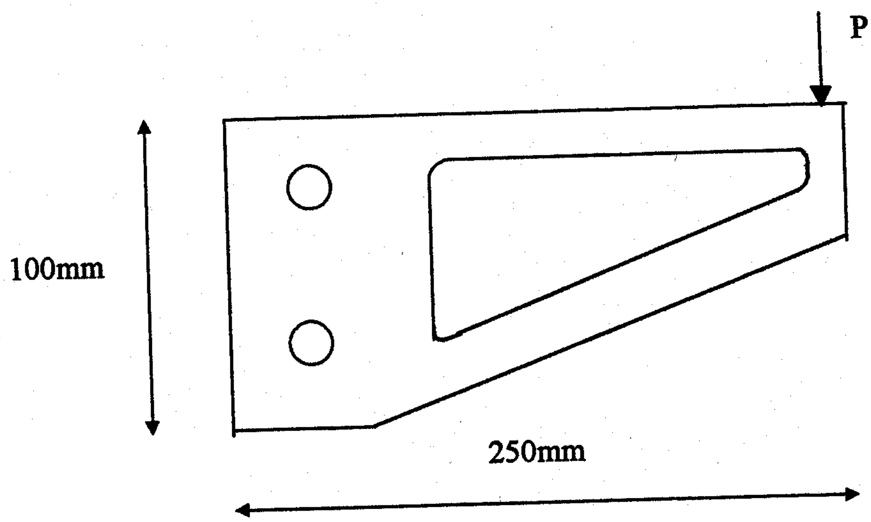
$$N_\alpha(L_1) = \prod_{i=1}^{\alpha}\left(\frac{nL_1-i+1}{i}\right) \qquad \alpha\ge 1$$

$$= 1 \qquad \alpha=0$$

Similarly for $N_\beta(L_2)$ and $N_\gamma(L_3)$.

Discuss the properties of this element with regard to continuity, completeness and geometric isotropy.

5. Give a detailed description of the procedure, which you would follow to carry out a finite element analysis of the support bracket shown in the figure using the ANSYS package. The steel bracket shown is fixed at the two small holes and is subject to a point force P= 10kN.
For steel, take E=200Gpa and $\nu = 0.3$. The plate thickness is 10mm and the support holes have a diameter of 20mm. Use assumed values for those plate dimensions not specified.



6. For knowledge based expert systems, discuss the following:
   (i) Knowledge Representation
   (ii) Search Strategies
   (iii) Applications in Civil Engineering

7. Planar seepage in a homogeneous, anisotropic medium is governed by the equation

$$k_x \frac{\partial^2 u}{\partial x^2} + k_y \frac{\partial^2 u}{\partial y^2} = 0$$

subject to the boundary conditions

$$u = \bar{u} \quad \text{on} \quad \Gamma_1$$

$$q = \frac{\partial u}{\partial n} = \bar{q} \quad \text{on} \quad \Gamma_2$$

where k_x and k_y are permeabilities and $\Gamma_1 + \Gamma_2 = \Gamma$=total boundary.

Derive the boundary element equations for this problem using constant elements.

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Lagrange Polynomials

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$$

Hermite Polynomials:

$$H_{01}^{1}(s) = 1 - 3s^2 + 2s^3$$
$$H_{11}^{1}(s) = Ls(s-1)^2$$
$$H_{02}^{1}(s) = s^2(3 - 2s)$$
$$H_{12}^{1}(s) = Ls^2(s-1)\ ; \ s = x/L$$

Integration by Parts:

$$\iiint_V u \frac{\partial v}{\partial x_i} dV = \iint_S (uv) n_i dS - \iiint_V v \frac{\partial u}{\partial x_i} dV$$

Finite Difference Formulae:

Deriv	Backward	Central	Forward
$\frac{df}{dx}$	$\frac{f_n - f_{n-1}}{h}$	$\frac{f_{n+1} - f_{n-1}}{2h}$	$\frac{f_{n+1} - f_{n-1}}{h}$
$\frac{d^2f}{dx^2}$	$\frac{f_n - 2f_{n-1} + f_{n-2}}{h^2}$	$\frac{f_{n+1} - 2f_n + f_{n-1}}{h^2}$	$\frac{f_{n+2} - 2f_{n+1} + f_n}{h^2}$
$\frac{d^3f}{dx^3}$	$\frac{f_n - 3f_{n-1} + 3f_{n-2} - f_{n-3}}{h^3}$	$\frac{f_{n+2} - 2f_{n+1} + 2f_{n-1} - f_{n-2}}{2h^3}$	$\frac{f_{n+3} - 3f_{n+2} + 3f_{n+1} - f_n}{h^3}$
$\frac{d^4f}{dx^4}$	$\frac{f_n - 4f_{n-1} + 6f_{n-2} - 4f_{n-3} + f_{n-4}}{h^4}$	$\frac{f_{n+2} - 4f_{n+1} + 6f_n - 4f_{n-1} + f_{n-2}}{h^4}$	$\frac{f_{n+4} - 4f_{n+3} + 6f_{n+2} - 4f_{n+1} + f_n}{h^4}$