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NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATION, 2000/2001

THIRD YEAR ELECTRONIC ENGINEERING

LINEAR CONTROL SYSTEMS

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Duration of Examination: *Three* hours
Instructions: Answer *five* questions

The following standard formulas are given and may be freely used :

$$\frac{M_p}{M_o} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad (\zeta \le 0.707)$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} \qquad (\zeta \le 0.707)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\omega_b = \omega_n\sqrt{(1-2\zeta^2)+\sqrt{(1-2\zeta^2)+1}}$$

$$T_r(0-95\%) \cong 3/\omega_b \qquad (\zeta > 0.4)$$

$$T_r(0-100\%) = \frac{\pi - \sin^{-1}\sqrt{1-\zeta^2}}{\omega_n\sqrt{1-\zeta^2}} \qquad (\zeta < 1)$$

$$Overshoot = 100\exp\left\{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right\}\% \qquad (\zeta < 1)$$

$$T_s(\pm 2\%) \le \frac{1}{\zeta\omega_n}\ln\left\{\frac{50}{\sqrt{1-\zeta^2}}\right\} \qquad (\zeta < 1)$$

$$T_s(\pm 5\%) \le \frac{1}{\zeta\omega_n}\ln\left\{\frac{20}{\sqrt{1-\zeta^2}}\right\} \qquad (\zeta < 1)$$

Ziegler-Nichols Rules :

Proportional : $K = 0.5K_c$

P+I control : $K = 0.45K_c$, $T_i = 0.83T_c$

PID : $K = 0.6K_c$, $T_i = 0.5T_c$, $T_d = 0.125T_c$

TABLE OF LAPLACE AND Z- TRANSFORMS

x(t)	X(s)	X(z)
$\delta(t)$	1	1
$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
t^2	$\frac{2}{s^3}$	$\frac{T^2 z(z+1)}{(z-1)^3}$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\frac{Tz e^{-aT}}{(z-e^{-aT})^2}$
$t^2 e^{-at}$	$\frac{2}{(s+a)^3}$	$\frac{T^2 z e^{-aT}(z+e^{-aT})}{(z-e^{-aT})^3}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
$1-e^{-at}(1+at)$	$\frac{a^2}{s(s+a)^2}$	$\frac{z}{z-1}\cdot\frac{z(1-e^{-aT}-aTe^{-aT})+(e^{-2aT}-e^{-aT}+aTe^{-aT})}{(z-e^{-aT})^2}$
$at-(1-e^{-at})$	$\frac{a^2}{s^2(s+a)}$	$\frac{z[(aT-1+e^{-aT})z+(1-e^{-aT}-aTe^{-aT})]}{(z-1)^2(z-e^{-aT})}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z[z-\cos(\omega T)]}{z^2-2z\cos(\omega T)+1}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{z e^{-aT}\sin(\omega T)}{z^2-2z e^{-aT}\cos(\omega T)+e^{-2aT}}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2-z e^{-aT}\cos(\omega T)}{z^2-2z e^{-aT}\cos(\omega T)+e^{-2aT}}$

1. It is required to design a controller for the plant shown in Fig.1, where sample measured values of $G_p(j\omega)$ are given in the following table:

ω (rad/s)	50	100	175	250
$G_p(j\omega)$	0 - j0.6	- 0.25 - j0.25	- 0.125 + j0	- 0.04 + j0.02

(a) Plot the above results onto cm graph paper and draw a smooth curve through the data points to give a polar plot (terminating at the origin). If simple proportional control is used (i.e. $G_c(s) = K$), specify the value of critical gain $K = K_c$ at which the system will become unstable and specify the period (T_c) of the unstable oscillations. **[5 marks]**

(b) If the gain is set at $K = 0.5K_c$ (the Ziegler-Nichols rule for proportional control), re-draw the polar plot to include account of the proportional controller. Specify the gain margin (in dB) and the phase margin (in degrees) of this design. Drawing on your experience, how much step response overshoot would you expect with this design? **[8 marks]**

(c) Phase-lead compensation is now added to the controller action, resulting in a modified controller with a transfer function specified by

$$G_c(s) = 4 \times \frac{5(s+50)}{s+250}$$

How much phase advance will the controller give at $\omega = 100$rad/s? Why is phase-lead compensation advantageous and why is phase lead compensation generally preferable to adding derivative action? **[7 marks]**

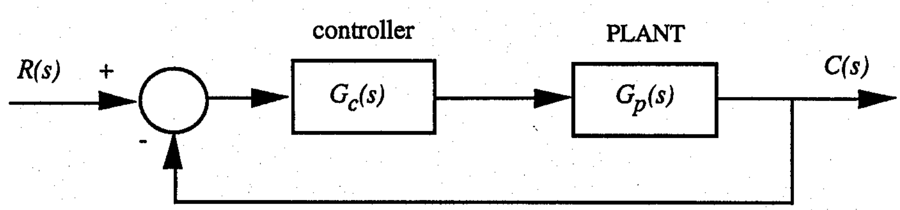

Fig.1

2. Fig.2 shows the block diagram of a control system with a P+I controller.

(a) Calculate and sketch the unit step response of the plant itself (i.e. the unit step response of the plant when separated from the the control loop). This is the response of the system under open-loop control. **[3 marks]**

(b) Briefly explain the advantage of using P+I control rather than simple proportional control **[2 marks]**

(c) Sketch the locus of the closed-loop poles as the gain K increases from zero to infinity. **[5 marks]**

(d) Use the angle condition to show that the point $s = -2 + j2$ lies on the root locus and use the magnitude condition to determine the design value of K needed to place closed-loop poles at $s = -2 \pm j2$. Specify the natural frequency ω_n and the damping ratio ζ associated with these pole locations. **[6 marks]**

(e) With the help of given formulas, or otherwise, draw an accurate sketch of the unit step response of the system when using the design value of K determined in part *(d)*. In particular, the rise time, overshoot, settling time, and final value should be clear from the sketch (but excessive accuracy is not required) **[4 marks]**

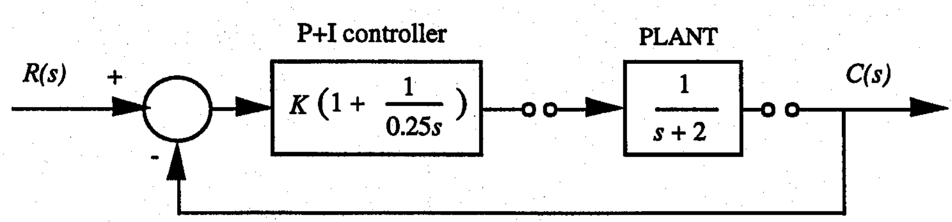

Fig.2

3. The control system of Fig.3 is fitted with a PID controller. The Ziegler-Nichols rules prescribe the following settings: $K = 10$, $T_d = 0.125$ seconds, $T_i = 0.5$ seconds.

Before closing the loop, open-loop tests were carried out with the controller set as above. These tests gave the following results :

Frequency	0.25Hz	0.5Hz	1Hz	2Hz	3Hz
Open-loop gain	26dB	11dB	0dB	-11dB	-22dB
Open-loop phase shift	-125°	-145°	-160°	-175°	-190°

Plot the results onto a Nichols Chart. Specify the gain margin (in dB) and the phase margin (in degrees). Assuming that closed-loop behaviour approximates to that of a standard second-order system, determine values for ζ and ω_n. Draw a pole-zero map showing the dominant pole locations. With the help of given formulas, or otherwise, sketch the expected nature of the unit step response showing the time scale involved and indicating the extent of any overshoot. Why will the final value be unity? **[16 marks]**

It is decided to reduce K from 10 to 6.4 and simultaneously increase T_d from 0.125 to 0.25 seconds and T_i from 0.5 to 1.0 seconds. For **5 bonus marks** (optional), show that the the new controller settings will shift the 1Hz data point from 0dB/-160° to 0dB/-117°. Show the new location of the 1Hz data point on the Nichols chart. Describe and explain how you would expect to the unit step response to change if the new controller settings are adopted. **[4 marks]**

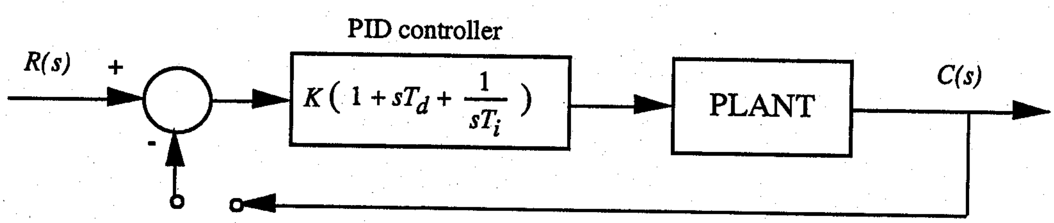

Fig.3

4. A particular digital control system has the z-plane representation shown in Fig.4.

(*a*) Determine the closed-loop transfer function $C(z)/R(z)$ in terms of the gain parameter K. **[4 marks]**

(*b*) If the gain is set at $K = 2.308$, show that the system will have poles at $z = 0.7$ and at $z = 0.8769 \pm j0.1848$. Hence show that the closed-loop transfer function of the system, with the given gain setting, will be:

$$\frac{C(z)}{R(z)} = \frac{0.04616(z-0.8)(z+0.6)}{(z-0.7)[(z-0.8769)^2+(0.1848)^2]}$$ **[4 marks]**

[*If you are unable to confirm this result, just use it in the next part of the question*]

(*c*) Draw the pole-zero map of the closed-loop transfer function in the z-plane. Given that the sampling interval is $T = 0.1$ seconds, map the poles and zeros from the z-plane into the s-plane. Use this mapping to obtain a sketch of the nature of the unit step response of the digital system, showing the time scale involved. Use the z-domain final-value theorem to confirm that the final value will be unity (allowing for rounding errors). **[12 marks]**

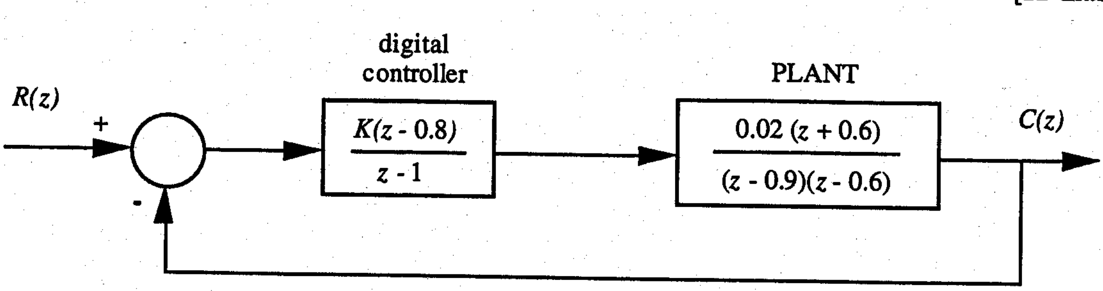

Fig.4

5. In the digital control system of Fig.5, the sampling interval is $T = 1$ second. The digital controller is programmed with the algorithm :

$$m(k) = m(k-1) + 0.25[e(k) + e(k-1)]$$

where $e(k) = r(k) - c(k)$. Assuming a zero-order DAC, represent the system in the z-domain and find the closed-loop transfer function $C(z)/R(z)$. Draw the pole-zero map of this transfer function in the z-plane. Proceed to map the poles into the s-plane. Sketch the expected nature of the unit step response of the digital control system, showing the time scale involved and the final value. Would you expect your sketch to be reasonably accurate (explain)?

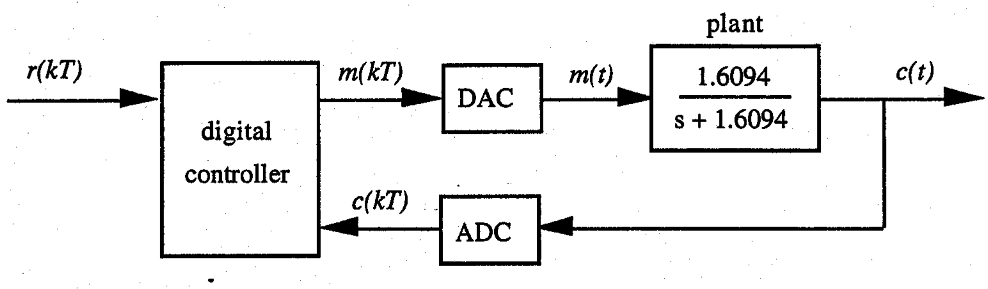

Fig.5

6. (*a*) Explain why a digital control system will be unstable if it has poles lying outside the unit circle in the z-plane. **[4 marks]**

(*b*) Show that the system shown in Fig.6 becomes unstable when $K = 1$. **[4 marks]**

(*c*) If K is set at $K = 0.5$, determine an expression in the z-domain relating the actuating signal $M(z)$ to the command signal $R(z)$, i.e. find an expression for $M(z)/R(z)$. **[4 marks]**

(*d*) Specify a recursive algorithm for calculating sample values of $m(k)$ from a succession of sample values of $r(k)$ for the case $K = 0.5$. **[4 marks]**

(*e*) Use the algorithm determined in part (*d*) to calculate the first five sample values of $m(k)$ for the case of a unit step input (i.e. $r(k) = 0$ for $k < 0$ and $r(k) = 1$ for $k \geq 1$). Sketch the actuating signal $m(t)$ which will appear at the plant input in this case, up to the instant $t = 5T$ (pay attention to the shape of the actuating signal). **[4 marks]**

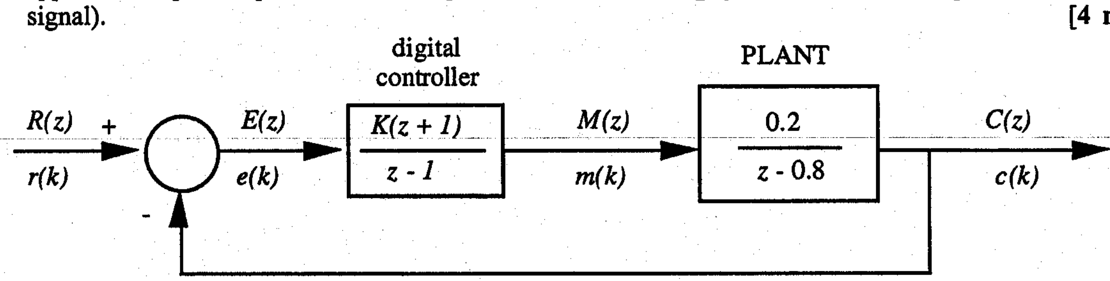

Fig.6

7. A digital controller is to be designed for for a certain closed-loop control system. The design specifications specify a step response overshoot not exceeding 20% and a ±2% settling time of about 2 seconds.

(*a*) Assuming that the closed-loop behaviour may be approximated by that of a second-order system, specify the pole locations in the s-plane corresponding to the design specifications. Choose a suitable sampling interval T for the digital design. Proceed to specify the desired (approximate) locations of the dominant closed-loop poles in the z-domain and show them on a pole-zero map. **[10 marks]**

(*b*) An analogue controller with a transfer function $G_c(s) = \dfrac{s+\alpha}{s}$ is found to meet the design specifications if $\alpha = 3$. If this transfer function is to be used as a basis for the required digital design with a sampling interval of $T = 0.2$ seconds, to what value should α be reduced as a pre-emulation measure to offset phase lag attributable to the DAC of the digital design (*hint*: compute the extra phase lag at $\omega = \omega_r = 2.61$ rad/s and adjust α to compensate for this). Derive the z-transform of a suitable digital controller using the bilinear transformation method. **[10 marks]**