GX552

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NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER 1 EXAMINATION, 2000/2001

3rd ELECTRONIC ENGINEERING
ERASMUS

E.M. THEORY AND APPLICATIONS II

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Duration of Examination: *two hours*

Instructions: Section **A** Answer *all* questions
Section **B** Answer *three* questions

Section A

(Attempt all questions in this section – 40 marks; 4 marks per question
Note that most results are assumed correct to two significant figures.)

A1. Given $\vec{a} = (2, -1, 3)$, $\vec{b} = (1, 3, 2)$ and $\vec{c} = (2, 1, -5)$, find the magnitude of the vector $3\vec{a} - 2\vec{b} + \vec{c}$.

(a) 10
(b) 13.4
(c) 15
(d) 8
(e) 21
(f) none of the above

A2. Given $\vec{a} = (1, -1, 3)$, $\vec{b} = (2, 1, 5)$ and $\vec{c} = (-1, 3, 2)$, find $\vec{a} \wedge (\vec{b} + \vec{c})$. [Note that $\vec{a} \wedge \vec{b}$ denotes the vector cross-product of ***a*** and ***b***]

(a) (19, 4, 5)
(b) (19, -4, -5)
(c) (19, 4, -5)
(d) (-19, 4, 5)
(e) (-19, -4, 5)
(f) none of the above

A3. Charges of -21μC and 16μC are placed 30cm apart. What is the total force on a charge of 9μC placed in the middle of the two charges?

(a) 2.7 N
(b) 33 N
(c) 4.5 N
(d) 133 N
(e) 18 N
(f) none of the above

A4. How far from an infinite line charge (charge density = $0.06\mu Cm^{-1}$) will a field strength of 10,000 NC^{-1} be felt?

(a) 0.05 m
(b) 0.11 m
(c) 0.23 m
(d) 0.27 m
(e) 0.33 m
(f) none of the above

A5. What is the strength of the E-field at a distance of 1.2m from an infinite plane with uniform charge density of $0.02\mu Cm^{-1}$?

(a) 125 NC^{-1}
(b) 250 NC^{-1}
(c) 785 NC^{-1}
(d) 942 NC^{-1}
(e) 1130 NC^{-1}
(f) none of the above

A6. An electric dipole, consisting of charges of magnitude 1.8nC separated by 5.8μm, is in a uniform electric field of 900 N/C. What is the potential energy difference between the dipole orientations parallel and anti-parallel to the E-field?

(a) 4.7 pJ
(b) 9.4 pJ
(c) 19 pJ
(d) 26 pJ
(e) 30 pJ
(f) none of the above

A7. Two parallel long straight wires, each of length 2m and carrying the same current of 5A, experience a repulsive force of 0.05N. What is the separation of the wires?

(a) 0.2 mm
(b) 0.3 mm
(c) 0.6
(d) 0.9 mm
(e) 1.2 mm
(f) none of the above

A8. A loop of diameter 0.4m is placed in a uniform B-field of 2.1T. Given that a current of 1.6A flows in the loop and that the axis of the loop is perpendicular to the direction of the B-field, find the torque on the loop.

(a) 0 Nm
(b) 0.13 Nm
(c) 0.42 Nm
(d) 1.7 Nm
(e) 2.1 Nm
(f) none of the above

A9. A conducting rod moves with velocity $5ms^{-1}$ perpendicular to a 3T uniform magnetic field. What is the length of the rod if an emf of 3.2V is induced?

(a) 4.3 cm
(b) 7.1 cm
(c) 16 cm
(d) 21 cm
(e) 37 cm
(f) none of the above

A10. The self-inductance of a solenoid of length 2m and radius 21mm is 0.5mH. How many turns per metre does it have?

(a) 55 turns
(b) 379 turns
(c) 536 turns
(d) 672 turns
(e) 758 turns
(f) none of the above

Section B

(Attempt 3 questions in this section – 20 marks each; candidates should note that marks may be lost if the solution to a question is not presented in a neat and orderly manner)

B 1. **(a)** Define the Electric Field and derive an expression for the Electric field strength, $\boldsymbol{E}$, due to a point charge, $+Q$. Find the $\boldsymbol{E}$ field strength due to uniform line charge, of length $2l$ and with uniform charge density of ρ_l Cm^{-1}, at a perpendicular distance of a from centre of the line. Deduce an expression for the $\boldsymbol{E}$ field strength due to an *infinite* line charge.

[12 marks]

(b) Deduce an expression for the $\boldsymbol{E}$ field strength due to an *infinite* charged plane with uniform charge density of ρ_A Cm^{-2}. In particular, show that this expression is independent of the perpendicular distance, a, from the plane. Pay attention to the symmetry of the problem.

[8 marks]

B 2. **(a)** Calculate the $\boldsymbol{E}$ field due to an electric dipole where the charges +Q and –Q are a distance l apart. How does this compare with the equivalent expression for $\boldsymbol{E}$ from a point charge?

[10 marks]

(b) Define electrical potential. Given 40 nC of charge is uniformly distributed around a circular ring of radius 2m find the potential, V, at a point on the axis which is 5m from the plane of the ring. Compare this with the case where all the charge is concentrated at the centre of the ring as a point charge.

[10 marks]

B 3. **(a)** Find the work done in moving a point charge $Q = 5$ μC from the origin to (2m, π/4, π/2) in spherical co-ordinates, in an external electric field, $\boldsymbol{E}$ such that:

$$\boldsymbol{E} = 5e^{-r/4}\hat{r} + \frac{10}{r\sin\theta}\hat{\phi} \text{ V/m?}$$

[10 marks]

(b) Derive a generalized expression for the energy stored in a system of N point charges. Find the stored energy in a system of four identical point charges, $Q = 4$ nC, located at the corners of a square with sides 1 m in length. What is the stored energy in the system when only two charges at opposite corners remain in place?

[10 marks]

B 4. **(a)** State, and explain briefly, the *Biot-Savart Law* and also *Ampere's Law*. Use sketches to help explain, and differentiate between, the concepts of both.

[6 marks]

(b) Find ***B*** at a distance x along the axis of a circular loop, radius a, of wire carrying a current I. Use the *Biot-Savart Law* and pay attention to the symmetry of the problem.

[7 marks]

(c) Find ***B*** at the centre of a square current loop with sides of length L and carrying a current I. Use the *Biot-Savart Law* and pay attention to the symmetry of the problem.

[7 marks]

B 5. **(a)** Explain the phenomenon of *Displacement Current*. Using Stokes' theorem show that the conduction and displacement currents are equal in a simple electrical circuit with an AC source and a capacitor.

[12 marks]

(b) State the point and integral form of each of Maxwell's equations in a table. Comment briefly on each equation, indicating the main physical phenomenon it describes. How do Maxwell's equations predict the existence of EM waves?

[8 marks]

Equations

(A) VECTORS

Scalar product: $a \cdot b = ab\cos\theta$ & $a \cdot b = b \cdot a$

Vector product: $a \wedge b = ab\sin\theta\,\hat{n}$ where a, b and $\hat{n}$ forms right-handed system.

$$a \wedge b = -b \wedge a$$
$$(a \wedge b)\cdot c = (c \wedge a)\cdot b = (b \wedge c)\cdot a$$

Differential operators:

$$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$$

$$\nabla \cdot E = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla \wedge E = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

(B) ELECTROSTATICS

$$\mu_0\varepsilon_0 = 1/c^2$$
$$\mu_0 = 4\pi\times10^{-7}\ \text{Hm}^{-1} \quad \text{(exactly)}$$
$$\varepsilon_0 = 8.854\times10^{-12}\,\text{Fm}^{-1}$$

Force between charges, $F = \frac{Q_1Q_2\hat{r}}{4\pi\varepsilon_0 r^2}$

Current, $I = \frac{dQ}{dt}$ (definition)

Potential difference between A and B, $\phi_B - \phi_A = -\int_A^B E\cdot dl$

Relation between E and ϕ, $E = -\nabla\phi$

$\oint E\cdot dl = 0$ (for an electrostatic field, ie. conservative field)

Gauss's theorem

$$\varepsilon_0\oint E\cdot dA = \sum Q_i = \int \rho\, dV$$

Electric dipole

Dipole moment, $p = Ql$ where l is the separation between $+Q$ and $-Q$.

Potential at $r >> l$, $\phi = \frac{p\cdot\hat{r}}{4\pi_0 r^2}$

In uniform E-field,

Force = 0 Couple is $G = p \wedge E$ Potential energy, $U = -p\cdot E$