GX560

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2000

B.E. DEGREE IN ELECTRONIC ENGINEERING

DIGITAL SIGNAL PROCESSING

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Duration of Examination : ***Three*** hours

Instructions: Answer **five** questions.
All questions carry equal marks.

1. (a) A discrete-time system is shown in Figure 1(a).

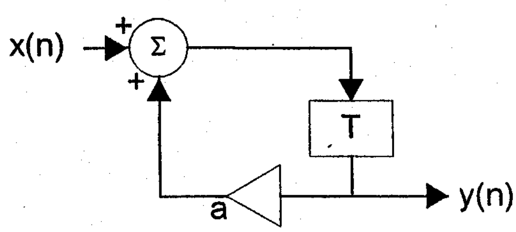
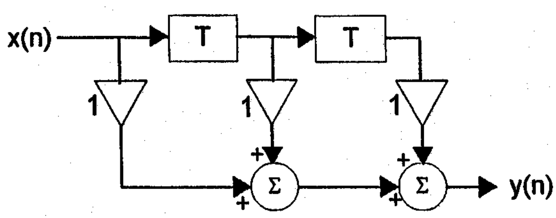


Figure 1(a)

Find the impulse response, $h(n)$, and the frequency response, $H(\theta)$, of the system. Hence, obtain an expression for the magnitude response.

(b) Two filter structures are shown in Figures 1(b) and 1(c):



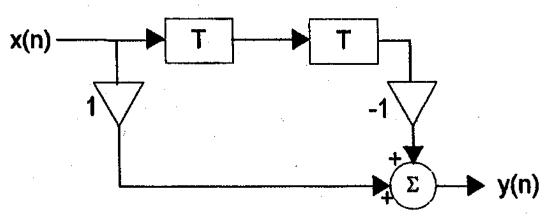


Figure 1(b) **Figure 1(c)**

Show that both filters have linear phase characteristics, and explain why linear phase is important in some signal processing applications.

2. (a) Give the equations for the Discrete Fourier Transform (DFT), and the inverse DFT, and explain how the magnitude spectrum of a signal may be obtained in this way. Explain what is meant by windowing, and why it is necessary in spectral analysis of discrete-time signals.

(b) A speech signal is sampled at a rate of 20 kHz, and a 1024-point DFT is computed on a frame by frame basis.
(i) What is the frequency resolution of the resulting spectrum?
(ii) What is the time resolution of the analysis?
(iii) Explain how the frequency resolution may be increased without affecting time resolution.

(c) The Fourier transform of the sequence $a^n u(n)$ is given by:

$$\frac{1}{1-ae^{-j\theta}}$$

Using this fact, find the output of a system with impulse response $h(n) = a^n u(n)$, when the sequence $b^n u(n)$ is input to the system (hint: recall the convolution property of the Fourier transform).

3. (a) Discuss the relative merits of FIR and IIR filters under the following headings:
(i) Computational complexity
(ii) Phase characteristics
(iii) Robustness, especially when implemented with finite wordlength

(b) Using the impulse-invariant transformation, transform the following analogue filter into a corresponding digital filter:

$$H(s) = \frac{2}{(s+1)(s+3)}$$

Hence, write the difference equation for the filter.

(c) Using the bilinear transformation, determine the transfer function of the digital equivalent of a first-order RC analogue filter, whose normalised (i.e. $\omega_c = 1$ rad/s) transfer function is given by:

$$H(s) = \frac{1}{s+1}$$

Assume a sampling frequency of 150 Hz, and a cutoff frequency of 30 Hz.

4. (a) Using the pole-zero placement method, design a digital band-pass filter with the following characteristics:

(i) Complete rejection of the input signal at DC and half the sampling frequency
(ii) A narow passband centred at 200 Hz
(iii) A 3 dB bandwidth of 20 Hz

The sampling frequency is 1 kHz. Also, sketch the pole-zero map of the filter.

(b) In biomedical signal processing applications (e.g. electrocardiogram measurements), interference from the 50 Hz mains voltage is common. Design a narrowband notch filter to reject this interference, where the sampling rate is 200 Hz. You may assume that a 3 dB notch bandwidth of 10 Hz is sufficiently narrow. Sketch the pole-zero map of the filter.

[cont'd]

4. (c) With the aid of a suitable example, explain the operation of frequency-sampling filters.

5. (a) The input signal to the system in Figure 2 is a band-pass signal whose spectrum extends from 500 Hz to 1 kHz. The filter $G(\theta_p)$ is a low-pass filter with cutoff frequency equal to $\pi/2$ (in digital frequency), while the carrier frequency of the modulator is 2 kHz.

(i) Determine the sampling frequency of each of the signals in figure 2.
(ii) Sketch $|X(\theta)|$ against θ and ωT,
(ii) Sketch $|P(\theta_p)|$, $|R(\theta_p)|$, and $|U(\theta_p)|$ against θ_p and ωT, where θ_p is the digital frequency variable for the upsampled signal.

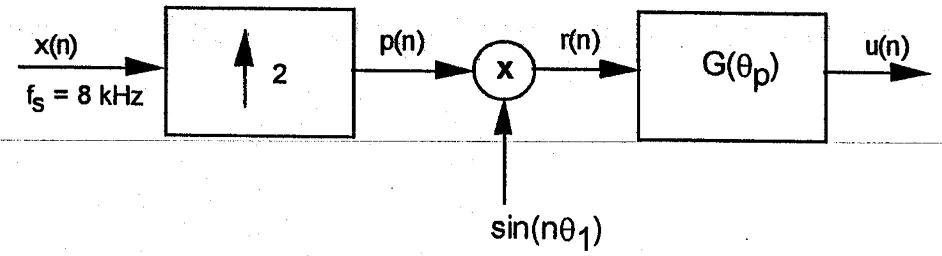


Figure 2

(b) If the Nyquist frequency (i.e. half the minimum sampling rate) for a certain analogue signal, $x_a(t)$, is Ω_s, determine the Nyquist frequency for the following signals derived from $x_a(t)$:

(i) $\dfrac{dx_a(t)}{dt}$
(ii) $x_a(2t)$
(iii) $x_a^2(t)$
(iv) $x_a(t)\cos(\Omega_0 t)$, i.e. $x_a(t)$ modulated by a sinusoid of frequency Ω_0.

6. (a) In the speech production model, a commonly used approximation for the impulse response of the glottal pulse-shaping filter is as follows:

$$g(n) = na^n, n \geq 0$$
$$g(n) = 0, n < 0$$

By obtaining the transfer function of this filter, choose a value of a such that the the magnitude response at DC is 60 dB higher than the response at half the sampling frequency.

(b) The autocorrelation function and the Average Magnitude Difference Function are to be used to extract the pitch of speech from a male speaker with a fundamental frequency of 150 Hz. Draw a rough sketch of the expected value for each of the two functions, indicating clearly the time differences between peaks and nulls. The sampling frequency is 8 kHz.

[cont'd]

6. (c) With the aid of equations and diagrams as appropriate, explain the following features of a speech signal:

(i) Zero-crossing Rate
(ii) Energy

Describe how these two features may be used for the task of end-point detection in a speech recognition system, and outline the circumstances under which endpoint detection might fail.

7. (a) With the aid of equations, describe the operation of the Least Mean Squares (LMS) algorithm, and discuss the stability and convergence speed of the algorithm.

(b) Describe the following low-complexity variants of the LMS algorithm:

(i) Signed-Error LMS
(ii) Signed-regressor LMS
(iii) Sign-Sign LMS

Give examples of where the Signed-regressor algorithm might be used, and explain how the hardware implementation of the LMS algorithm may be further reduced by judicious choice of the step size.

Table of the z transforms of a few useful sequences

		Sequence	z-Transform
1.	Unit sample	d(n)	1
		d(n-k)	z^{-k}
2.	Unit step	u(n)	z/(z-1)
3.	Exponential	a^n u(n)	z/(z-a)
4.	Sinusoidal	$\sin(\theta_o n)\, u(n)$	$\dfrac{z \sin\theta_o}{z^2 - 2z\cos\theta_o + 1}$
		$\cos(\theta_o n)\, u(n)$	$\dfrac{z^2 - z\cos\theta_o}{z^2 - 2z\cos\theta_o + 1}$
5.	Unit ramp	nu(n)	$\dfrac{z}{(z-1)^2}$
6.	$1 + z^{-1} + z^{-2} + z^{-3} + \dots\dots\dots + z^{-(N-1)} = \dfrac{1-z^{-N}}{1-z^{-1}}$		

7. $n\,x(n)$ $\qquad -z \dfrac{dX(z)}{dz}$