GX576

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

Summer Examinations 2000

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B.Sc. (Honours) Degree Examination(4BS)

Mathematics Option:
*IE428 - '**Quality Control**'*

Dr. Wright,
Prof. O'Kelly,
Dr. Sheil.

Time allowed: ***Two*** hours

Attempt ***three*** questions

Cambridge Statistical Tables supplied.
Mathematical('log') Tables available

Q1.

(i) Consider an $\bar{x}$ control chart having an upper control limit only, at the value A. Let μ, σ^2 represent the current process mean and variance for the measured characteristic X and let the subgroup size be n.
Show that the probability that individual $\bar{x}$ values fall below A is given, at least approximately, by $\Phi(\frac{A-\mu}{\sigma/\sqrt{n}})$, where $\Phi(.)$ is the Standard Normal cdf. [4 marks]

If the target value and process standard deviation for X are 201, 0.5 respectively, and $n = 9$:
(a) what value must A have, so that the probability of *false alarms* is 0.002; [4 marks]
(b) if the process has been incorrectly centered at 205.2, how much time would you expect to elapse before this error is signalled by the control chart - you may assume that the control chart is updated hourly? [5 marks]

(ii) $\bar{x}$, R charts based on subgroups of size 3, 4 or 5 will be used to monitor a number of machines and subprocesses throughout a plant. Design a generic form upon which to record data and plot the charts. This form should be capable of customisation for use by relevant personnel in the areas concerned. [7 marks]

Q2.

How are CUSUM charts visually interpreted? [2 marks]

If $S_j = \sum_{i=1}^{j}(x_i - m)$, show that x_j may be expressed: $x_j = m + S_j - S_{j-1}$. [2 marks]

Hence deduce that $\frac{1}{n}\sum_{i=r+1}^{n} x_i = m + \frac{S_n - S_r}{n}$. [3 marks]

What use can be made of this result, when analysing CUSUM plots? [2 marks]

The scrap rate associated with the output of a foundry is under investigation. Based on past experience, management has set a target of 20% for the average rate of scrap. Past experience has also shown that when the foundry is operating normally, the standard deviation of day-to-day variation in the scrap rate is approximately 0.5%. Scrap rates(%) for the past 20 days of production were as follows: 19.2, 21.4, 21.3, 20.3, 21.0, 20.0, 22.0, 21.0, 20.1, 18.2, 19.7, 19.5, 19.4, 19.8, 20.5, 20.7, 20.6, 21.2, 20.3, 21.1. Analyze this data with respect to the target set by management. [11 marks]

Q3.

Small Components Ltd. produce aluminium components. The company operates two eight hour shifts per day and production operators monitor their own work.
A dimension of a component which will be produced on one of the company's machines, carries the specification 202±4. In a carefully controlled prototype run, 20 such components were produced from this machine. Both operators associated with the machine, twice measured each of the 20 parts for the dimension of interest, using the designated gauge - the order in which the 80 measurements involved were conducted, was randomised. The following table presents the readings recorded.

Part No.	Operator 1		Operator 2	
1	202	202	203	201
2	203	201	202	202
3	201	200	199	200
4	206	207	206	208
5	200	201	199	200
6	199	199	200	199
7	202	201	201	201
8	202	200	200	201
9	201	201	199	201
10	204	203	203	203
11	201	202	200	201
12	200	199	200	198
13	202	201	203	201
14	203	202	203	203
15	203	201	204	202
16	205	206	205	207
17	201	200	200	201
18	204	204	204	202
19	202	203	201	202
20	202	201	201	203

(a) Quantify *gauge repeatability* and *gauge reproducability* and comment [Note: $d_2 = 1.128$]. [12 marks]

(b) Estimate *gauge capability.* [3 marks]

(c) Comment on machine/process capability. [5 marks]

Q4.

When would you employ an *np-chart*, a *p-chart*? [2 marks]

A company produces cable and harness for the computer and automotive industries. From time to time problems arise with a process on the line. As a result, there is an increase in the number of units which exhibit a lack of electrical integrity: when in control the process yields 98% good units.
You have been asked to institute an *np-chart* on this process. You wonder how large the sample size n should be, and have been advised to choose n so that there is a high probability of finding at least one defective in the sample even when the process is in control.

(i) Why is this a good idea? [2 marks]

(ii) Show that, in this case, n = 114 is the smallest value for n which ensures that the probability of finding at least one defective in the sample, when the process is in control, is greater than 0.9? You may assume that the number of defectives in the sample follows a Binomial distribution. [4 marks]

(iii) Determine a centerline and upper control limit for your np-chart. [4 marks]
Why are lower control limits frequently omitted from such charts? [2 marks]

(iv) Sketch the OC-curve for the control chart derived at (iii) above. [6 marks]
Note: For 'large' n, $Bi(n, p) \rightarrow Poi(np)$ when p is 'small'.

Q5.

Define each of the following, in the general context of Acceptance Sampling.
acceptance number, *operating characteristic(OC)*, *rectifying inspection* [3 marks]

Show, using standard notation, that when a single attributes sampling plan is used in conjunction with rectifying inspection, the average amount of inspection per lot is

$I(\theta) = N - (N - n).P(\theta)$ [3 marks]

Hence, deduce that the average outgoing quality(AOQ) which results is

$A(\theta) = \frac{(N-n)}{N}\theta P(\theta)$ [2 marks]

A single sampling plan with $n = 100$, $c = 2$ is used for receiving inspection.
Lots contain 1000 items and 'rejected' lots are screened/rectified by the vendor.

(ii) Sketch the AOQ curve for this plan and estimate the AOQL. [8 marks]

(iii) For a single sampling plan with c = 0, how large should n be, if the AOQL is to be the same as that for the $n = 100$, $c = 2$ plan above? [4 marks]