Gx643

OLLSCOIL NA hÉIREANN
The National University of Ireland

National University of Ireland, Galway.

Michaelmas Examinations, 2000/01

Fourth Year Mechanical and Biomedical Engineering Examination

FINITE ELEMENT METHODS IN ENGINEERING ANALYSIS

Professor J.J. O'Connor
Professor J.F.McNamara

Attempt Five Questions
Time allowed: 3 Hours

1. A uniform prismatic rod of length L and mass ρA per unit length hangs vertically from a fixed point. A concentrated mass of ρAL is attached at the lower end. The vertical displacement of a point on the rod distant x from the fixed end is u(x).

The potential energy functional governing the elastic deformations of the rod is given as

$$\Pi(u) = \int_0^L \left\{ \frac{EA}{2}\left(\frac{du}{dx}\right)^2 - \rho g A u \right\} dx - \rho g A L u(L)$$

where E is Young's modulus for the rod material.

(a) Develop the finite element equations for the rod using the functional Π (u). *(10)*

(b) Model the rod as a simple two-noded truss element and solve for the end displacement u(L) in terms of the given physical paramenters. *(5)*

(c) Comment briefly on the accuracy of your solution. *(5)*

2. A composite wall is composed of two sections with properties as given in Figure 2. Convection occurs on the left hand side whereas the right hand side is held at 20^0C.

You are required to develop a one-dimensional finite element solution for the temperature profile across the wall using linear interpolation functions and one element for each of the wall sections. You are not required to numerically solve the final equations except to indicate the application of the boundary condition for $T = 20^0$C.

(20)

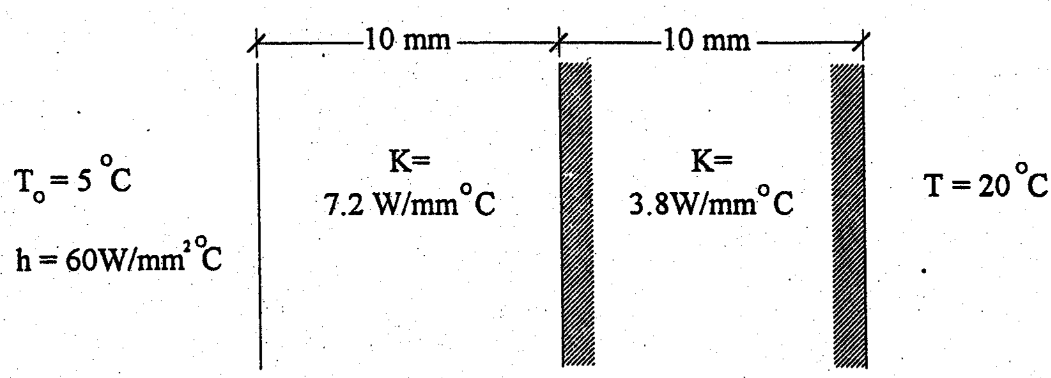

FIGURE 2

The general functional for this class of problem is given as

$$\Pi = \int_0^L \frac{1}{2}\left\{K\left(\frac{dT}{dx}\right)^2 - 2QT\right\}A\,dx + \int_{S_1} qT\,ds + \int_{S_2} \frac{h}{2}(T - T_o)^2\,ds$$

where symbols have their usual meanings.

3. The results of a finite element analysis of an elastic membrane under plane stress assumptions generated the nodal displacement components as listed for the simple triangular element shown in Figure 3. These nodal displacements are due to a temperature rise of 50^0C in addition to specified pressure loads.

(a) Compute the total strains in the element due to the nodal deformations. *(10)*

(b) Compute the thermal strains due to the change in temperature of 50^0C. *(5)*

(c) Compute the strains due to pressure loads only. *(5)*

Note: The derivatives of area with respect to Cartesian coordinates are given as follows:

$$\frac{\partial Li}{\partial x} = (y_j - y_k)/2A; \frac{\partial Li}{\partial y} = (x_k - x_j)/2A$$

where A is the area of the triangle and i, j and k are cyclic.

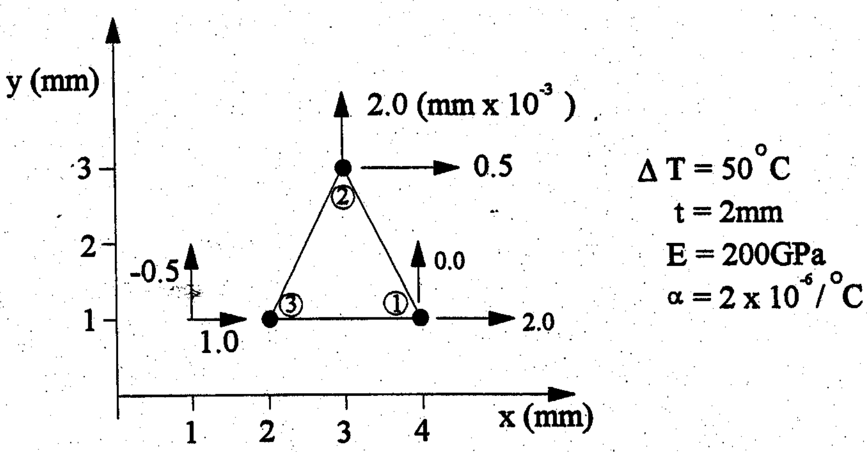

FIGURE 3

4. The transverse and axial displacements of the two-noded beam-column element shown in Figure 4(a) are written, respectively, as follows;

$$v(x)=N_1(x)v_1+N_2(x)\theta_1+N_3(x)v_2+N_4(x)\theta_2$$

$$u(x)=N_5(x)u_1+N_6(x)u_2$$

where

$N_1(x) = (1+2x/_L)(1-x/_L)^2$
$N_2(x) = x(1-x/_L)^2$
$N_3(x) = (x/_L)^2(3-2x/_L)$
$N_4(x) = x(x/_L)(-1+x/_L)$
$N_5(x) = 1-x/_L$
$N_6(x) = x/_L$

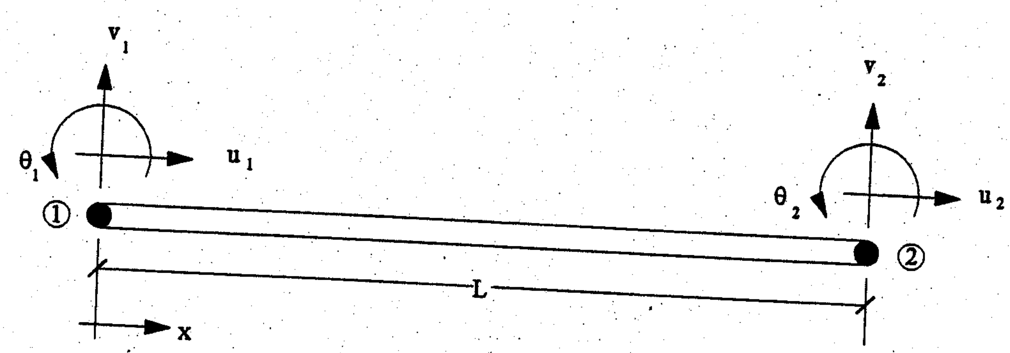

FIGURE 4(a)

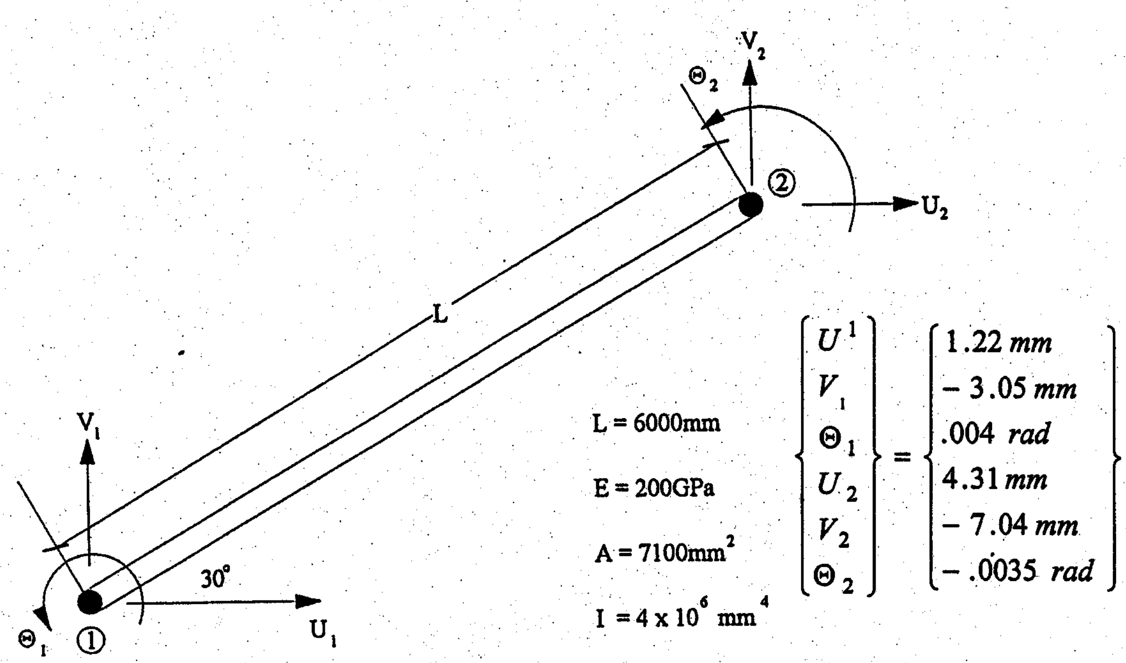

FIGURE 4(b)

(a) Compute the curvature and bending moment at the midpoint of the single element shown in Figure 4(b) due to the given global deformations.

(12)

(b) Compute the axial strain and axial force for the element in Figure 4(b) due to the given global deformations.

(8)

5. The nodal interpolation functions for the 8-noded quadrilateral isoparametric element are given as follows:

$N_i(\xi,\eta)=\frac{1}{4}(1+\bar{\xi})(1+\bar{\eta})(\bar{\xi}+\bar{\eta}-1)$ for the four corner nodes at $\xi_i=\pm1$ *and* $\eta_i=\pm1$

$N_i(\xi,\eta)=\frac{1}{2}(1-\xi^2)(1+\bar{\eta})$ for the midside nodes at $\xi_i=0$ *and* $\eta_i=\pm1$

$N_i(\xi,\eta)=\frac{1}{2}(1-\eta^2)(1+\bar{\xi})$ for the midside nodes at $\eta_i=0$ and $\xi_i=\pm1$ where $\bar{\xi}=\xi\xi_i$ *and* $\bar{\eta}=\eta\eta_i$

Compute the thermal load vector $\int_S q\underline{N}^T\,ds$ due to heat conduction on the side with nodes 4, 7 and 3 as shown in Figure 5. Apply a simple check to show that your numerical result is correct.

(20)

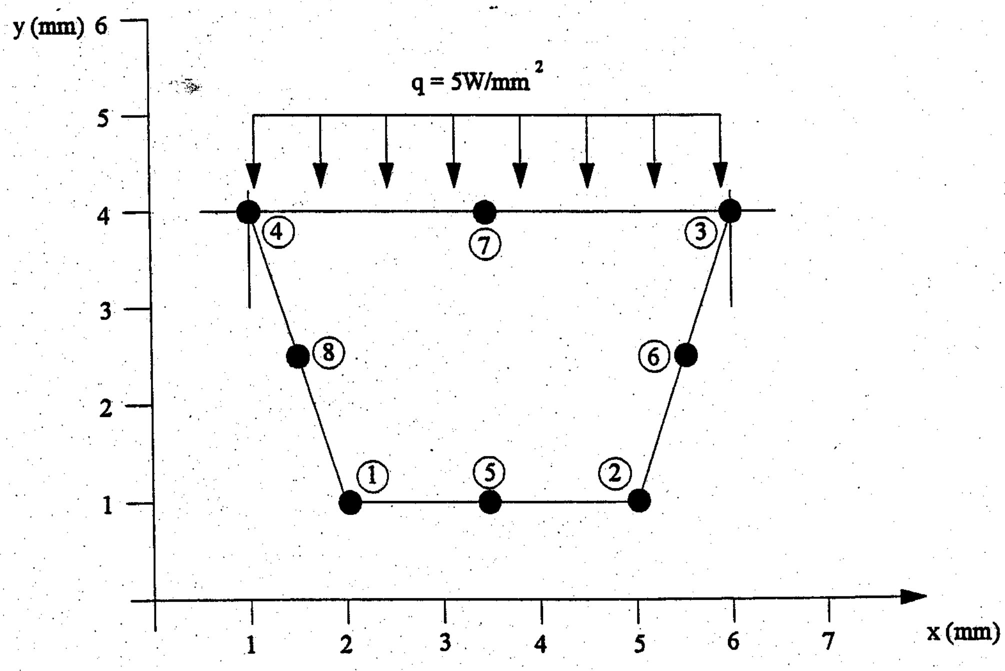

FIGURE 5

6(a) Derive the general form of the Jacobian for the three-noded triangular element with nodes at each vertex only.

(6)

(b) Evaluate the Jacobian and its determinant for the triangle shown in Figure 6 and indicate how you may readily check the accuracy of you result.

(6)

(c) Calculate the partial derivatives

$\dfrac{\partial L_1}{\partial x}$ and $\dfrac{\partial L_2}{\partial y}$ for the triangle of Figure 6.

(8)

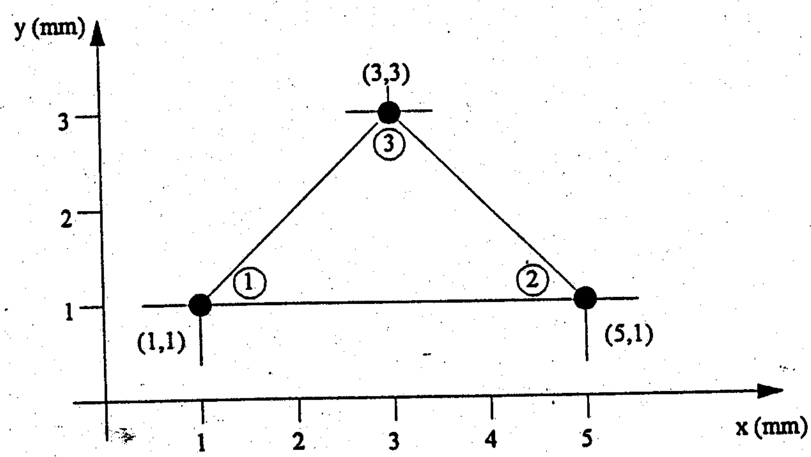

FIGURE 6