GX645

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The National University of Ireland

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M.Eng.Sc. Examination (Mechanical)

MECHANICS OF COMPOSITE MATERIALS

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Attempt Q.1 and One Other Question
<u>All Questions are worth 50 marks</u>

1(a) An unsymmetric $[0/90]$ laminate is being tested in tension. Longitudinal strain gauges are mounted on both the front and back surfaces as sown in Figure 1(a). The ply thickness is 0.75mm and the laminate width is 20mm.

At an applied load of 10.0KN the outputs of the strain gauges are as follows:

Front Gauge: $\varepsilon = 30 \times 10^{-3}$ (=3%)
Back Gauge: $\varepsilon = 10 \times 10^{-3}$ (=1%)

(i) What is the Young's Modulus of the laminate in the x-direction?
(ii) What will the radius of curvature of the laminate be at an applied load of 15KN?

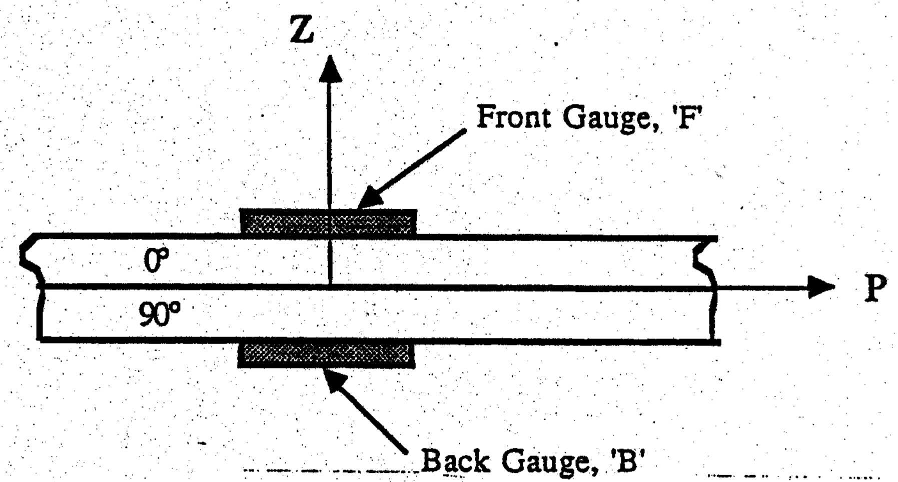


Figure 1(a)

1(b) Show that the in-plane shear modulus, G_{12} can be determined from data recorded from the 45°C tensile test, Figure 1(b).

$$G_{12} = \frac{E_X\left(\pm 45^0 \ specimen\right)}{2\left(1+\nu_{XY}\right)\left(\pm 45^0 \ specimen\right)}$$

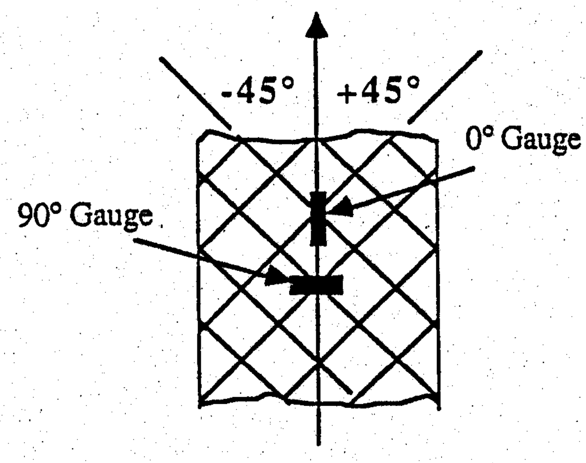


Figure 1(b)

1(c) A 25⁰ specimen of unidirectional composite material is heated from room temperature to 250°C in an oven. A strain-gauge rosette, mounted at room temperature, shown in Figure 1(c) reads the following data:

$$\varepsilon_A = 0.38\%$$
$$\varepsilon_B = 0.29\%$$
$$\varepsilon_C = 1.34\%$$

Calculate the thermal expansion coefficients, α_1 and α_2 of the material.

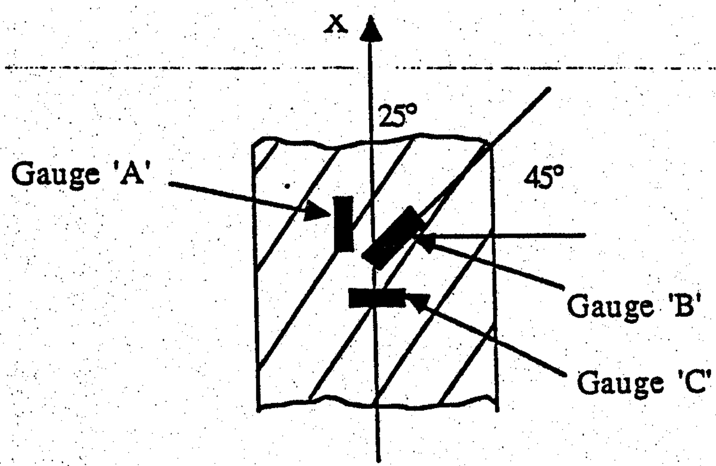


Figure 1(c)

1(d) A 40^0 specimen of unidirectional material is loaded in equal biaxial tension. Given the material properties below, use the following failure criteria to predict the value of biaxial tensile stress which causes failure. In cases (i) and (ii) also identify the failure mode.

(i) Maximum Strain Criterion
(ii) Maximum Stress Criterion
(iii) Tsai-Wu Failure Criterion

$E_L = 77.0$GPa ; $E_T = 5.2$GPa; $G_{LT} = 3.8$GPa ; $\nu_{LT} = 0.35$

$X_1^T = 1830$ MPa ; $X_1^C = 1052$MPa; $X_2^T = 50$MPa ;

$X_2^C = 70$MPa; $X_6 = 75$MPa

$Y_1^T = 2.0$ %; $Y_1^C = 1.3$%; $Y_2^T = 0.96$ % , $Y_2^C = 1.35$%

$Y_6 = 1.9$%

(50)

2. A $\left[0^0/\pm 60\right]_{2S}$ laminate of glass/PEI is cooled from its stress-free temperature of 200^0C to room temperature (20^0C). The laminate is then loaded in biaxial tension until first-ply failure occurs.

The following loading is applied:

$$\sigma_x^* = \sigma_0, \ \sigma_y = 0.5\sigma_0$$

Given the material properties below, calculate the values of σ_0 that will cause first-ply failure in each ply.

Where will the first failure occur in the laminate, and at what value of σ_0?

Given:

$E_L = 134$GPa , $E_T = 8.9$ GPa , $\nu_{LT} = 0.28$, $G_{LT} = 5.1$ GPa

$\alpha_1 = 2.0 \times 10^{-6} / {}^0$C, $\alpha_1 = 2.0 \times 10^{-6} / {}^0$C,

$Y_1^T = 1.83$% , $Y_1^C = 0.3$% , $Y_2^T = 0.21$% , $Y_2^C = 0.99$%

$Y_6 = 1.55$%

(50)

3. A 2.4 mm thick S-Glass/Epoxy $[90^0/0^0]_S$ plate is tested in uniform bonding in the X-direction. Given the material properties below, calculate the value of M_x which causes

(i) first-ply failure

(ii) subsequent, and ultimate failure

Given: $E_L = 55.0$ GPa, $E_T = 14.0$GPa, $G_{LT} = 3.4$GPa, $\nu_{LT} = 0.3$

$X_1^C = X_1^T = 1100$ MPa, $X_2^T = X_2^C = 120$ MPa

$X_6 = 100$ MPa

NOTE:

1. Use the Maximum Stress Failure criterion
2. Assume the 90^0 plies unload with modulus $E_T = -3.0$ GPa

(50)

ME 505 - MECHANICS OF COMPOSITE MATERIALS - EQUATIONS

[Q] IN TERMS OF ENG. CONSTANTS

$$Q_{11} = \frac{E_1}{[1 - \nu_{12}^2(E_2/E_1)]} \cdot Q_{12} = \frac{\nu_{12}E_2}{[1 - \nu_{12}^2(E_2/E_1)]}$$

$$Q_{22} = \frac{E_2}{[1 - \nu_{12}^2(E_2/E_1)]} \cdot Q_{66} = G_{12}$$

EXPANSIONAL STRESS RESULTANTS

$$N_x^E = \int_{-h/2}^{h/2} (\epsilon_1^{E^k} \overline{Q}_{11}^k + \epsilon_2^{E^k} \overline{Q}_{12}^k + \epsilon_6^{E^k} \overline{Q}_{16}^k) dz$$

$$N_y^E = \int_{-h/2}^{h/2} (\epsilon_1^{E^k} \overline{Q}_{12}^k + \epsilon_2^{E^k} \overline{Q}_{22}^k + \epsilon_6^{E^k} \overline{Q}_{26}^k) dz$$

$$N_{xy}^E = \int_{-h/2}^{h/2} (\epsilon_1^{E^k} \overline{Q}_{16}^k + \epsilon_2^{E^k} \overline{Q}_{26}^k + \epsilon_6^{E^k} \overline{Q}_{66}^k) dz$$

STIFFNESS TRANSFORMATIONS

$$\overline{Q}_{11} = Q_{11}m^4 + 2m^2n^2(Q_{12} + 2Q_{66}) + Q_{22}n^4$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)$$

$$\overline{Q}_{16} = [(Q_{11} - Q_{12} - 2Q_{66})m^2 - (Q_{22} - Q_{12} - 2Q_{66})n^2]mn$$

$$\overline{Q}_{22} = Q_{22}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{11}n^4$$

$$\overline{Q}_{26} = -[(Q_{22} - Q_{12} - 2Q_{66})m^2 - (Q_{11} - Q_{12} - 2Q_{66})n^2]mn$$

$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12})m^2n^2 + Q_{66}(m^2 - n^2)^2$$

COMPLIANCE TRANSFORMATIONS

$$\overline{S}_{11} = S_{11}m^4 + (2S_{12} + S_{66})m^2n^2 + S_{22}n^4$$

$$\overline{S}_{12} = (S_{11} + S_{22} - S_{66})m^2n^2 + S_{12}(m^4 + n^4)$$

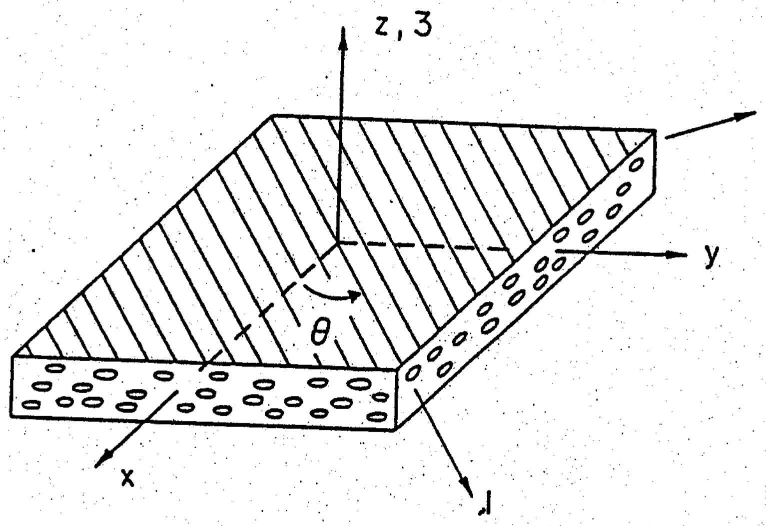
$$\overline{S}_{16} = -[(2S_{11} - 2S_{12} - S_{66})m^2 - (2S_{22} - 2S_{12} - S_{66})n^2]mn$$

$$\overline{S}_{12} = S_{22}m^4 + (2S_{12} + S_{66})m^2n^2 + S_{11}n^4$$

$$\overline{S}_{26} = [(2S_{22} - 2S_{12} - S_{66})m^2 - (2S_{11} - 2S_{12} - S_{66})n^2]mn$$

$$\overline{S}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})m^2n^2 + S_{66}(m^4 + n^4)$$

Transformation of Stress and Strain



$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{bmatrix}$$

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

Tsai-Wu Failure Theory

$$F_1\sigma_1 + F_2\sigma_2 + F_6\tau_{12} + F_{11}\sigma_1{}^2 + F_{22}\sigma_2{}^2 + F_{66}\tau_{12}{}^2 + 2F_{12}\sigma_1\sigma_2 = 1$$

where

$$F_1 = 1/X_1^T - 1/X_1^C \qquad F_{11} = 1/(X_1^T X_1^C)$$

$$F_2 = 1/X_2^T - 1/X_2^C \qquad F_{22} = 1/(X_2^T X_2^C)$$

$$F_6 = 1/S_6^+ - 1/S_6^- \qquad F_{66} = 1/(S_6^+ S_6^-)$$

$$F_{12} = -1/(2\sqrt{X_1^T X_2^T X_1^C X_2^C})$$