GX658

OLLSCOIL NA hÉIREANN
The National University of Ireland

National University of Ireland, Galway

Summer Examinations, 2000

BE Degree (Mechanical) Examination
BE Degree (Biomedical) Examination

ADVANCED MECHANICAL ANALYSIS AND DESIGN

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Attempt 5 Questions.
Time Allowed: 3 Hrs.
Tables 8.1, 9.4, 11.1 and 11.2 from Burr & Cheatham are attached.

1(a) Name 6 material constants that relate to an isotropic linear elastic material and describe the types of deformations and formulae in which they are most suitably used. How many constant values are required to specify all 6? *(4)*

(b) Establish the compliance equations (strains as functions of stresses) for an isotropic linear elastic material in 3 dimensions. Write the final result in matrix form. Assume a Cartesian (x, y, z) coordinate system. Clearly state the assumptions of plane stress, plane strain and generalised plane strain. Simplify the compliance equations for the cases of plane stress and plane strain. *(8)*

(c) Clearly state the assumptions of axial-symmetric deformations. Assume a cylindrical polar coordinate system (r, θ, z). Derive the following expressions for the radial strain, ε_r, and the tangential strain, ε_t, in terms of the radial displacement u.

$$\varepsilon_r = \frac{du}{dr}$$

$$\varepsilon_t = \frac{u}{r}$$

What do these expressions illustrate regarding the relationship between displacementand strain in the axial-symmetric case? *(8)*

2. Water flows from an elevation of 500 m through a circular pipe that ends with 180° bend into the casing of a water turbine. The bend is shown in Figure 2. Derive expressions for the meridian stress, σ_m, and the tangential stress, σ_t, in the bend section at points (1), (2) and (3) , in terms of R_0, r, t and pressure p. For the dimensions given, estimate the required steel thickness for the bend under conditions of static fluid pressure. The specific weight of water is 10 kN/m^3 and you may assume a factor of safety of 2.0 based on a tensile yield stress of 250 MPa. *(20)*

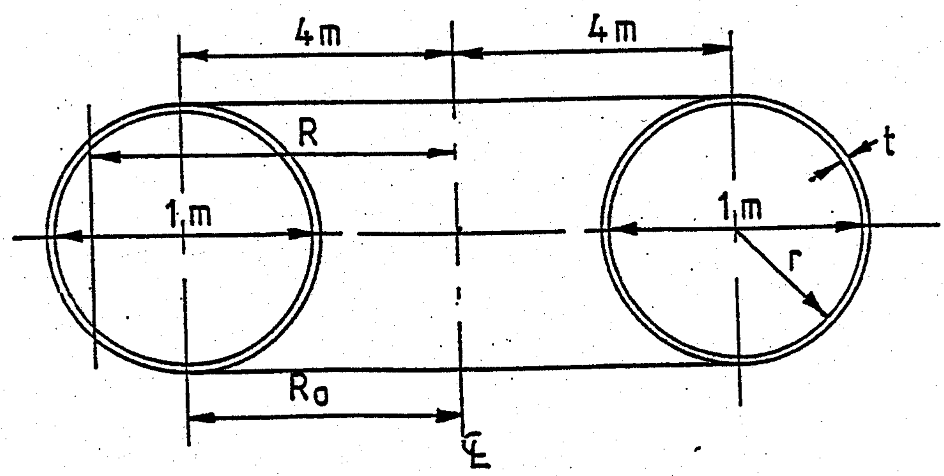


Figure 2

3(a) Consider a compound thick-walled cylinder with open ends made of two layers, with an internal radius m, an interface radius n and an external radius q. The cylinder is subject to an internal pressure p_i. Given that there is interface pressure p_f between the layers, derive the following expressions for the maximum value of shear stress in the inner cylinder:

$$\tau_{max}^{inner} = \frac{-p_f n^2}{n^2 - m^2} + \frac{p_i q^2}{q^2 - m^2},$$

and in the outer cylinder:

$$\tau_{max}^{outer} = \frac{p_f q^2}{q^2 - n^2} + \frac{p_i m^2 q^2}{(q^2 - m^2) n^2}.$$ *(10)*

(b) In designing a compound cylinder, where both layers are made from the same material, a maximum allowable shear stress of τ_0 is assumed. Also, based on a fixed internal radius, m, it is desirable to minimise material, i.e., minimise the

external radius. It is found that this is achieved when $n^2 = mq$. Substitute this result, and the maximum allowable shear stress, into the result of (a) and derive the following expressions for the radii q and n.

$$q = \frac{m}{1 - p_i / 2\tau_0}$$
$$n = m\sqrt{\frac{1}{1 - p_i / 2\tau_0}} \qquad (10)$$

4(a) Consider the shafts loaded as shown in Figure 4a. All four shafts are simply supported at their ends. For each of the cases, sketch the shear force and bending moment diagrams, and state whether or not they are statically determinate. *(8)*

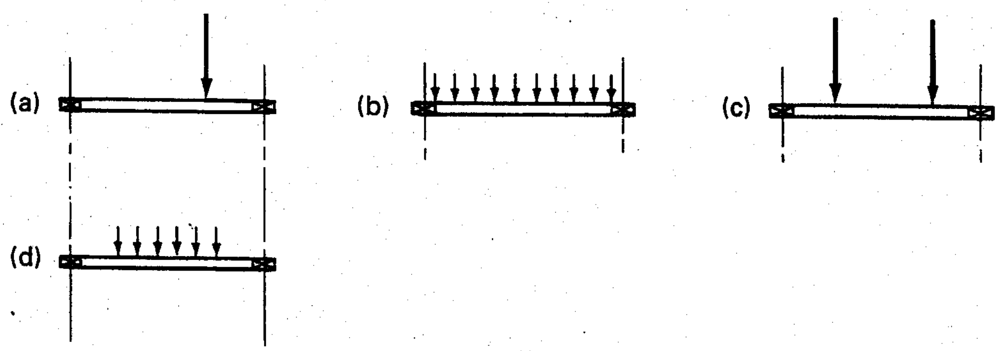


Figure 4a

(b) The steel camshaft of Figure 4b for a fuel injection pump has a distance between bearing centres of 266 mm, divided into seven equal spaces by the centre lines of the six cams. The lift is 9.2 mm, the plunger diameter is 13 mm and the maximum injection pressure is 55 MPa. Assume that the resulting force acts on only one cam at a time. Ignoring the stiffening effect of the cams, determine the shaft diameter required to limit the deflection at any cam to 0.25 mm. E = 207 GPa and the second moment of area of a circular shaft is $\pi d^4/64$, where d is the shaft diameter. Determine the maximum stress in the shaft. (12)

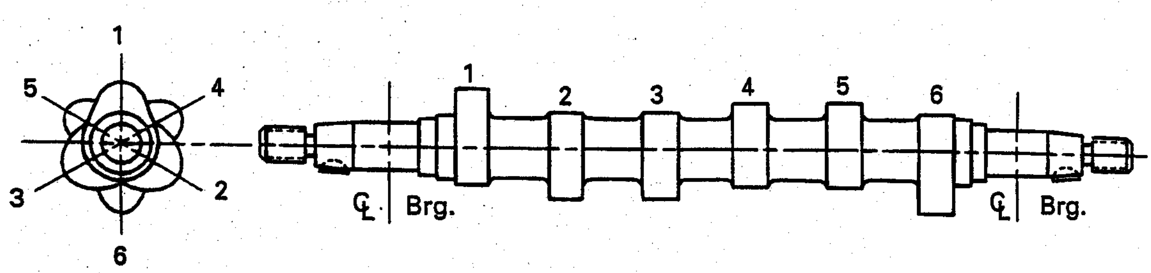


Figure 4b

5(a) Determine an expression for the displacement of a gap in a split ring when the ring is subjected to uniformly distributed pressure p. *(16)*

(b) Assuming that the density of the ring is ρ write down an expression for the displacement of the gap when the ring is rotated at an angular speed ω. *(4)*

6(a) The pivot shown in Figure 6 has a rounded point with a radius of 0.5 mm. It rests on a flat surface and supports a force P. Both surfaces are of hardened steel and the allowable compressive stress at the point of contact is 2000 MPa. What is the force P if it is due to a mass of 204 kg? What length is required for the pivot so that the allowable compressive stress is not exceeded? Assume that the acceleration due to gravity is 9.81 m/sec^2. *(10)*

(b) Determine the maximum compressive stress at a location 2 mm above the contact point in Figure 6 and comment on its value. (Hint: use Table 11.1). *(10)*

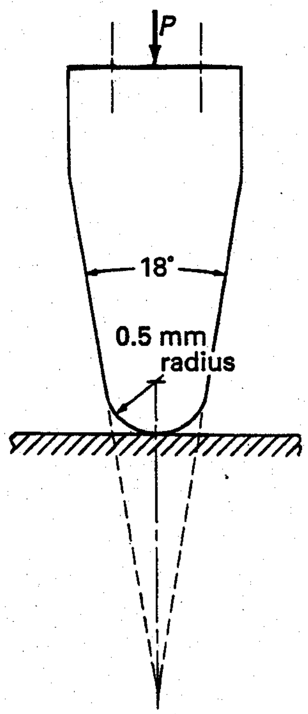


Figure 6

7(a) A T section for a curved beam has the dimensions shown in Figure 7a. Show that the neutral surface has a radius, r_n, given by

$$r_n = \frac{b_i(r_j - r_i) + b_o(r_o - r_j)}{b_i \ln(r_j / r_i) + b_o \ln(r_o / r_j)} .$$ *(6)*

(b) The C hook or "hairpin" hook shown in Figure 7b is used in rolling mills for lifting and transporting large coils of wire and rods. The hook of a crane is put into one of the notches of the top piece or "eye". Its location is selected according to the number and position of the coils, such that the hook tilts slightly counterclockwise. With the dimensions shown and with a stress of 140 MPa and a uniform distribution of coils along the 1 metre length, what would be the web width b of a T cross section for a capacity of 2500 kg? Assume that the acceleration due to gravity is 9.81 m/sec^2. *(14)*

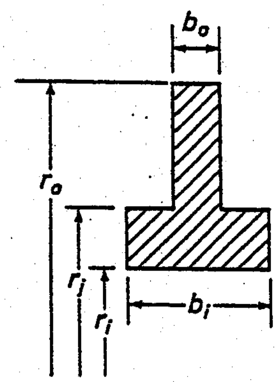


Figure 7a

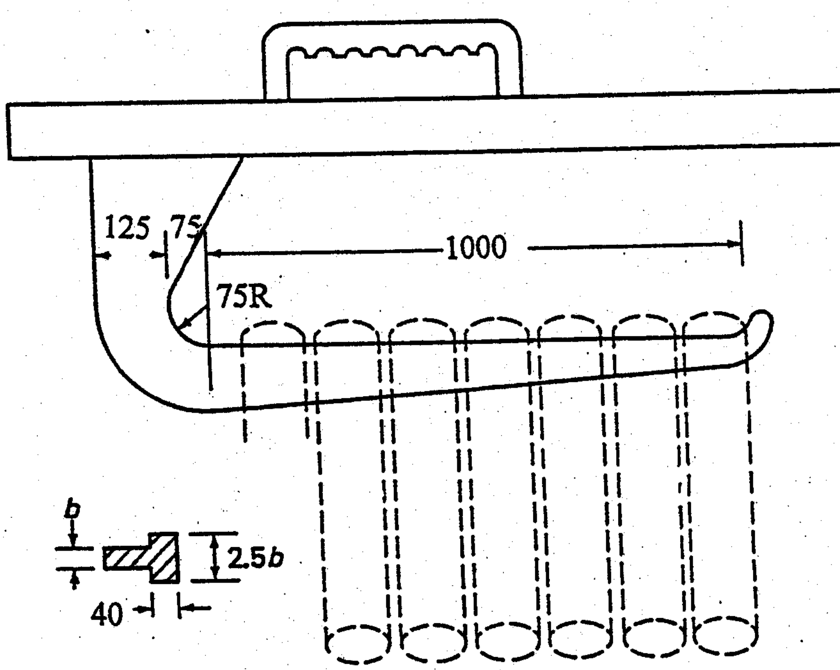


Figure 7b

TABLE 8.1. Cylinders of Uniform Length—Loading, Boundary Conditions, Stresses and Displacements. ρ = density, ν = Poisson's ratio, E = modulus of elasticity

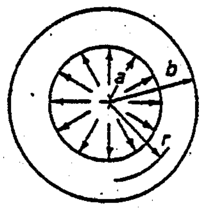
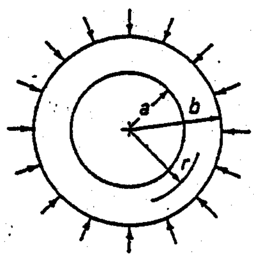
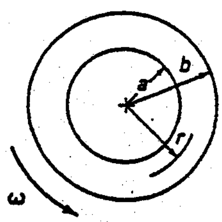
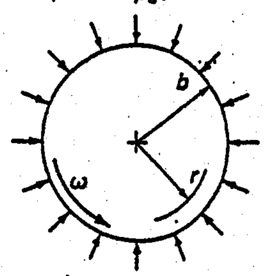
Loading	Boundary Conditions	Stresses and Displacement
(1) Internal pressure p_i 	at $r = a$, $\sigma_r = -p_i$ at $r = b$, $\sigma_r = 0$	$\sigma_r = -p_i \dfrac{a^2}{b^2-a^2}\left(\dfrac{b^2}{r^2}-1\right)$, max $\sigma_r = -p_i$ at $r = a$ $\sigma_t = p_i \dfrac{a^2}{b^2-a^2}\left(\dfrac{b^2}{r^2}+1\right)$, max $\sigma_t = p_i \dfrac{b^2+a^2}{b^2-a^2}$ at $r = a$ $u = p_i \dfrac{r}{E}\dfrac{a^2}{b^2-a^2}\left[(1-\nu)+(1+\nu)\dfrac{b^2}{r^2}\right]$
(2) External pressure p_o 	at $r = 0$, $\sigma_r = 0$ at $r = b$ $\sigma_r = -p_o$	$\sigma_r = -p_o \dfrac{b^2}{b^2-a^2}\left(1-\dfrac{a^2}{r^2}\right)$, max $\sigma_r = -p_o$ at $r = b$ $\sigma_t = -p_o \dfrac{b^2}{b^2-a^2}\left(1+\dfrac{a^2}{r^2}\right)$, max $\sigma_t = -p_o \dfrac{2b^2}{b^2-a^2}$ at $r = a$ $u = -p_o \dfrac{r}{E}\dfrac{b^2}{b^2-a^2}\left[(1-\nu)+(1+\nu)\dfrac{a^2}{r^2}\right]$
(3) Thin uniform disk. Rotation ω. 	at $r = a$, $\sigma_r = 0$ at $r = b$, $\sigma_r = 0$	$\sigma_r = \rho\omega^2 \dfrac{3+\nu}{8}\left(b^2+a^2-\dfrac{a^2b^2}{r^2}-r^2\right)$ max $\sigma_r = \rho\omega^2 \dfrac{3+\nu}{8}(b-a)^2$ at $r = \sqrt{ab}$ $\sigma_t = \rho\omega^2 \dfrac{3+\nu}{8}\left(b^2+a^2+\dfrac{a^2b^2}{r^2}-\dfrac{1+3\nu}{3+\nu}r^2\right)$ max $\sigma_t = \dfrac{\rho\omega^2}{4}\left[(3+\nu)b^2+(1-\nu)a^2\right]$ at $r = a$ $u = \rho\omega^2 \dfrac{r}{E}\dfrac{(3+\nu)(1-\nu)}{8}\left(b^2+a^2+\dfrac{1+\nu}{1-\nu}\dfrac{a^2b^2}{r^2}-\dfrac{1+\nu}{3+\nu}r^2\right)$
(4) Solid, thin uniform disk. Rotation ω and external pressure p_o. 	at $r = 0$, $u = 0$ at $r = b$, $\sigma_r = -p_o$	$\sigma_r = -p_o + \rho\omega^2 \dfrac{3+\nu}{8}(b^2-r^2)$ max $\sigma_r = -p_o + \rho\omega^2 \dfrac{3+\nu}{8} b^2$ at $r = 0$ $\sigma_t = -p_o + \rho\omega^2 \dfrac{3+\nu}{8}\left(b^2-\dfrac{1+3\nu}{3+\nu}r^2\right)$ max σ_t = max σ_r at $r = 0$ $u = \dfrac{r}{E}(1-\nu)\left\{-p_o+\dfrac{\rho\omega^2}{8}\left[(3+\nu)b^2-(1+\nu)r^2\right]\right\}$

TABLE 9.4. Slopes and deflections of cantilever and simply supported beams (signs by inspection)

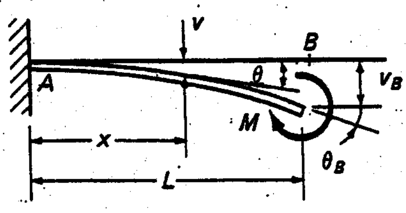
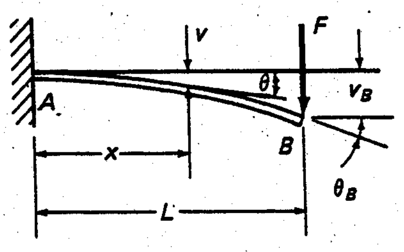
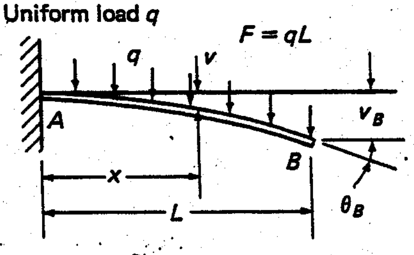
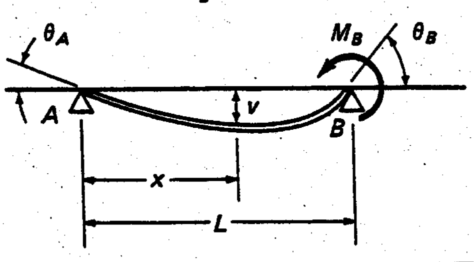
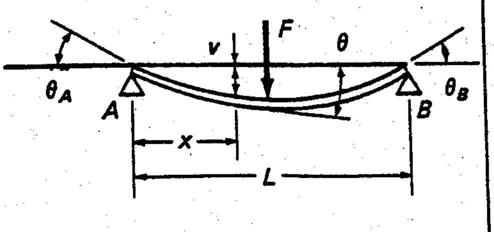
Loading and Diagram	Slope	Deflection
Cantilever Beams		
1. End moment M 	$\theta = \frac{Mx}{EI}$ $\theta_B = \frac{ML}{EI}$	$v = \frac{Mx^2}{2EI}$ $v_B = \frac{ML^2}{2EI}$
2. End force F 	$\theta = \frac{Fx(2L-x)}{2EI}$ $\theta_B = \frac{FL^2}{2EI}$	$v = \frac{Fx^2(3L-x)}{6EI}$ $v_B = \frac{FL^3}{3EI}$
3. Uniform load q 	$\theta = \frac{Fx(3L^2-3Lx+x^2)}{6EIL}$ $\theta_B = \frac{FL^2}{6EI}$	$v = \frac{Fx^2(6L^2-4Lx+x^2)}{24EIL}$ $v_B = \frac{FL^3}{8EI}$
Simply Supported Beams		
4. End moment M_B 	$\theta_A = \frac{M_B L}{6EI}$ $\theta_B = \frac{M_B L}{3EI}$	$v = \frac{M_B x(L^2-x^2)}{6EIL}$ $v_{max} = 0.0642 \frac{M_B L^2}{EI}$ at $x = 0.578L$
5. Central force F 	$\theta = \frac{F(L^2-4x^2)}{16EI}$ $(x \leq L/2)$ $\theta_A = \theta_B = \frac{FL^2}{16EI}$	$v = \frac{F(3L^2x-4x^3)}{48EI}$ $(x \leq L/2)$ $v_{max} = \frac{FL^3}{48EI}$ at $x = L/2$

TABLE 9.4. *(continued)*

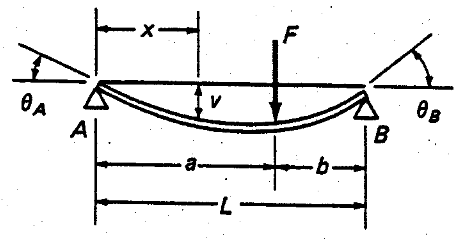
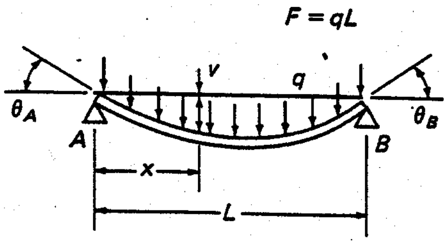
Loading and Diagram	Slope	Deflection
	Simply Supported Beams	
6. Off-center force F 	$\theta_A = \frac{Fb(L^2 - b^2)}{6EIL}$ $\theta_B = \frac{Fa(L^2 - a^2)}{6EIL}$	$v = \frac{Fbx(L^2 - b^2 - x^2)}{6EIL}$ $(x \leqslant a)$ $v_{max} = \frac{0.0642Fb(L^2 - b^2)^{3/2}}{EIL}$ at $x = 0.578\sqrt{L^2 - b^2}$
7. Uniform load q 	$\theta = \frac{q(L^3 - 6Lx^2 + 4x^3)}{24EI}$ $\theta_A = \theta_B = \frac{FL^2}{24EI}$	$v = \frac{Fx(L^3 - 2Lx^2 + x^3)}{24EIL}$ $v_{max} = \frac{5FL^3}{384EI}$ at $x = L/2$

TABLE 11.1. Concentrated and Distributed Surface Forces

Notation: Z-axis normal to surface, z = depth, r = radial distance as sketched, P = force applied, l = length of contact line, P/l = force per unit length, σ = normal stress, τ = shear stress, s = resultant stress, w = surface deflection, E = modulus of elasticity, ν = Poisson's ratio, $\eta = (1-\nu^2)/E$

Limitations: Dimensions of the body must be much larger than the locally affected portion.

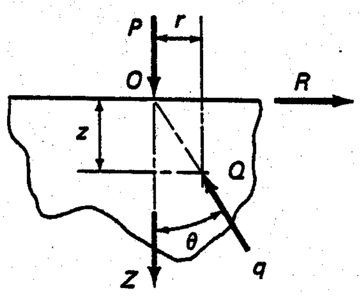
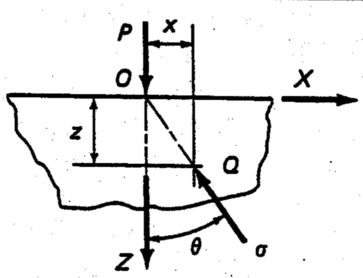
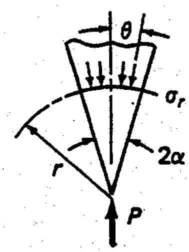
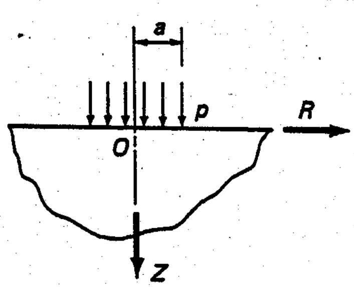
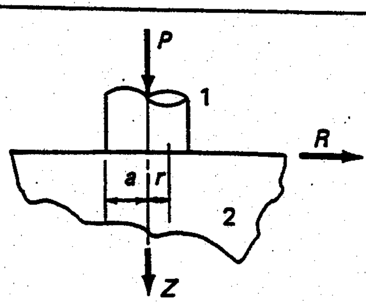
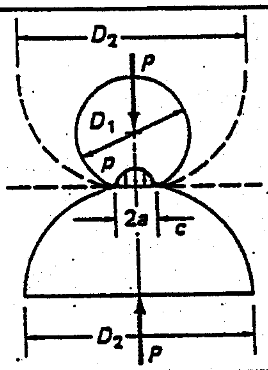
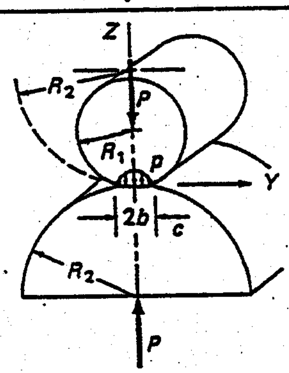
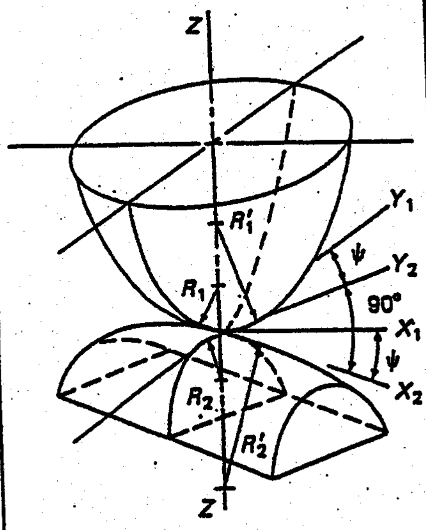
<table>
<tr><th>Loading Case</th><th>Pictorial</th><th>Stresses and Deflections</th></tr>
<tr><td>1. Point</td><td>
</td><td>$q=\sqrt{\sigma_z^2+\tau_{rz}^2}=\frac{3}{2\pi}\frac{P\cos^2\theta}{(r^2+z^2)}$
$w=\frac{1-\nu^2}{\pi E}\frac{P}{r}$ at surface</td></tr>
<tr><td>2. Line</td><td>
</td><td>$\sigma=\frac{2}{\pi}\frac{(P/l)\cos\theta}{\sqrt{x^2+z^2}}$</td></tr>
<tr><td>3. Knife edge or pivot</td><td>
</td><td>$\sigma_r=\frac{(P/l)\cos\theta}{r(\alpha+\frac{1}{2}\sin 2\alpha)}$</td></tr>
<tr><td>4. Uniform distributed load p over circle of radius a.</td><td>
</td><td>With $\nu=0.3$, at point O
$\sigma_z=-p,\quad \sigma_r=\sigma_\theta=-0.8p$
and $w_{max}=\frac{2(1-\nu^2)pa}{E}=2\eta pa$
$\tau_{max}=0.33p$ at $z=0.638a$</td></tr>
<tr><td>5. Rigid cylinder ($E_1 > E_2$)</td><td>
</td><td>$(\sigma_z)_{z=0}=-p=-\frac{P}{2\pi a\sqrt{a^2-r^2}}$
$w=\frac{(1-\nu^2)P}{2E_2a}=\frac{\eta_2 P}{2a}$</td></tr>
</table>

TABLE 11.2. Contact between Two Elastic Bodies

Notation: a = major semiaxis of ellipse of contact (radius of circle of contact between spheres), b = minor semiaxis of ellipse, also half-width of rectangle of contact between parallel cylinders, where l = length of contact; P = load, P/l = load per unit length of parallel cylinders, p_0 = maximum pressure on surface, σ_c = compressive stress, σ_t = tensile stress, τ = shear stress, δ = approach of the bodies as a whole; E = modulus of elasticity, ν = Poisson's Ratio, and

$$\eta_1 = \frac{1-\nu_1^2}{E_1} \quad \text{and} \quad \eta_2 = \frac{1-\nu_2^2}{E_2}$$

(For steel, $\eta = 0.0303 \times 10^{-6}$ in²/lb or $\eta = 4.40 \times 10^{-6}$ mm²/N)

Loading case	Pictorial	Area, Pressure, Approach
1. Spheres or Sphere and Plane If the surface of the body 2 is concave (dash lines), take D_2 negative, if plane, take $D_2 = \infty$.		$a = 0.721\,[P(\eta_1+\eta_2)D_1D_2/(D_1+D_2)]^{1/3} = c/2$ $p_0 = 1.5\,P/\pi a^2 = 1.5\,p_{avg} = -(\sigma_c)_{max}$ $\max \tau = \frac{1}{3}\,p_0$, at depth $0.638\,a$ $\max \sigma_t = (1-2\nu)\,p_0/3$, at radius a $\delta = 1.04\,[(\eta_1+\eta_2)^2P^2\,(D_1+D_2)/D_1D_2]^{1/3}$
2. Cylindrical Surfaces with Parallel Axes If the surface of body 2 is concave (dash lines) take R_2 negative. If plane, take $R_2 = \infty$ except for δ.		$b = 1.13\sqrt{(P/l)\,(\eta_1+\eta_2)\,R_1R_2/(R_1+R_2)} = c/2$ $p_0 = 2P/\pi bl = 1.273\,p_{avg} = -(\sigma_c)_{max}$ If $\nu = 0.30$ $\max \tau = 0.304\,p_0$, at depth $0.786\,b$ If $\eta_1 = \eta_2 = \eta$ $\delta = 0.638\,(P/l)\,\eta\left[\frac{2}{3} + \ln\frac{2R_1}{b} + \ln\frac{2R_2}{b}\right]$
3. General Case At point of contact, Z-axis is the common normal. Minimum and maximum numerical values of the radii of curvature are respectively R_1 and R_1' for body 1, R_2 and R_2' for body 2. Angle between planes containing curvatures $1/R_1$ and $1/R_2$ is ψ. A radius is negative if the surface is concave in its plane.		$b = \beta\left[\frac{3P(\eta_1+\eta_2)}{4(B+A)}\right]^{1/3}$ and $a = b/\kappa$ where β, κ, and λ are obtained from Fig. 11.6 and $B+A = \frac{1}{2}\left[\frac{1}{R_1}+\frac{1}{R_1'}+\frac{1}{R_2}+\frac{1}{R_2'}\right]$ $B-A = \frac{1}{2}\left\{\left[\frac{1}{R_1}-\frac{1}{R_1'}\right]^2+\left[\frac{1}{R_2}-\frac{1}{R_2'}\right]^2 + 2\left[\frac{1}{R_1}-\frac{1}{R_1'}\right]\left[\frac{1}{R_2}-\frac{1}{R_2'}\right]\cos 2\psi\right\}^{1/2}$ at $x = y = z = 0$ $p_0 = 1.5\,P/\pi ab = 1.5\,p_{avg} = -(\sigma_c)_{max}$ $\sigma_x = -2\nu p_0 - (1-2\nu)\,p_0\,\frac{b}{a+b}$ $\sigma_y = -2\nu p_0 - (1-2\nu)\,p_0\,\frac{a}{a+b}$ $\delta = \lambda\,[P^2\,(\eta_1+\eta_2)^2\,(B+A)]^{1/3}$