GX744

OLLSCOIL NA hÉIREANN
GAILLIMH

NATIONAL UNIVERSITY OF IRELAND
GALWAY

SEMESTER 1 (WINTER) EXAMINATIONS 2000-2001

3rd Year B.Sc. Unit EP311 : Light and Electromagnetism

Dr. J.M. Woolsey
Prof. R.M. Redfern
Dr. G. Gillanders

Time allowed : TWO hours.	Answer THREE questions

Q.1 Answer both (a) and (b).

(a) Show how the information contained in Maxwell's Equations may be reduced to the equations

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma \frac{\partial \vec{E}}{\partial t} + \frac{1}{\varepsilon}\vec{\nabla}\rho$$

$$\nabla^2 \vec{B} = \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} + \mu\sigma \frac{\partial \vec{B}}{\partial t}$$

(Hint: Begin by taking $\vec{\nabla}\times(\vec{\nabla}\times\vec{E})$ and $\vec{\nabla}\times(\vec{\nabla}\times\vec{B})$, *i.e.* the curls of the Maxwell's equations, respectively.)

Show how these equations may be interpreted to predict electromagnetic waves traveling through a vacuum at c, the speed of light. (Note: $\varepsilon_0 = 8.85 \times 10^{-12}$ F.m^{-1}, $\mu_0 = 4\pi \times 10^{-7}$ H.m^{-1}, $c = 3\times10^8$ m.s^{-1}) What is the relationship (in terms of magnitude and direction) between the electric and magnetic fields in these waves?

(b) Discuss the concept of a Poynting vector and derive an expression for it in terms of the electric and magnetic components of an electromagnetic wave. How is the intensity of an electromagnetic wave related to the Poynting vector?

Q.2 Answer all three parts.

(a) Show how two linearly polarized waves with orthogonal planes of polarization can be combined to form another linearly polarized wave.

(b) Describe the theory behind half wave plates. Show how a half wave plate may be used to rotate the plane of polarization of a monochromatic light beam through 90°.

(c) What minimum thickness of mica is required to form a half wave plate for light of wavelength 589 nm. The refractive indices for mica are: $n_o = 1.599$, $n_e = 1.594$. What effect would a plate 50% this thickness have on linearly polarized incident light?

Q.3 Draw a labelled diagram of a Michelson interferometer and briefly explain the function of its various components.

A Michelson interferometer, illuminated with monochromatic light, is adjusted to give circular fringes with a dark fringe at the centre of the fringe pattern. Beginning with the expression $m\lambda = 2d\cos\theta$, where the various symbols have their usual meaning, derive an expression for the angle at which the pth dark fringe from the centre occurs.

Describe the fringe pattern obtained with this interferometer and discuss how it changes as d is decreased. Briefly discuss the fringe pattern obtained if one of the mirrors in the interferometer is adjusted so that the mirrors are no longer perpendicular to each other.

Looking into the beam splitter of a Michelson interferometer, illuminated with light of wavelength 632.8 nm, you see straight-line fringes separated by 1.30 mm. Calculate the angle by which the planes of its two mirrors deviate from mutual perpendicularity.

Note: $$\cos\theta \approx 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} \cdots$$

Q.4. Show that the Fraunhofer Diffraction pattern that results from a line of N harmonic oscillators all in phase with one another is described by the expression

$$I = I_o \frac{\sin^2 \frac{N\delta}{2}}{\sin^2 \frac{\delta}{2}} \quad , \quad \delta = ka\sin\theta$$

where the various symbols have their usual meaning.

Note: $$1 + x^a + x^{2a} + \ldots + x^{(N-1)a} = \frac{1 - x^{Na}}{1 - x^a}$$

Sketch, and briefly discuss, the diffraction pattern obtained for $N = 6$. Derive a formula for the angles, θ, at which the principal maxima occur in such diffraction patterns.

The sodium D-lines (wavelengths 589.0 nm and 589.6 nm) are barely resolved in the first order when a 2 mm wide section of a grating is illuminated with light from a sodium lamp. (a) How many lines per mm are ruled on this grating? (b) What is the angular separation of the sodium D-lines in the second order. (Hint: Resolution of a grating is given by $\lambda / \Delta\lambda = mN$ where the various symbols have their usual meanings.)

Q.5 Derive an expression for the relationship between the half angle of the cone of acceptance of a step index optical fibre, the refractive index of the medium from which light enters the fibre (n_i), the refractive index of the fibre core (n_f), and the refractive index of the cladding (n_c). Hence define the term *numerical aperture* of a fibre.

A ray is incident on the end face of an optical fibre at an angle of incidence θ_i. l, the path length travelled by the ray through the fibre is given by

$$l = \frac{n_f L}{\sqrt{n_f^2 - n_i^2 \sin^2 \theta_i}}$$

where L is the length of the fibre. Beginning with this expression, derive an expression for the time delay between rays travelling the longest and shortest practical paths along an optical fibre of length L.

A step index optical fibre has a core of refractive index 1.500 and a cladding of refractive index 1.485. (i) Calculate the numerical aperture of this fibre. (ii) What is the maximum angle of incidence for which light entering from air will be transmitted effectively along the fibre? (iii) Calculate the time delay per kilometre between rays travelling the longest and shortest practical paths along this fibre. (iv) Calculate the maximum frequency at which you would reasonably send digital pulses down 5 km of this fibre. (Note: $c = 3\text{x}10^8$ m.s^{-1}, refractive index of air = 1.000)

Appendix: EP311 - Light and Electromagnetism - Formula Sheet

3-d differential wave equation (cartesian coords.) $\nabla^2 \psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$

Vector calculus relationship

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A}$$

Maxwell's Equations - Integral form

$$\oint_C \vec{E}.d\vec{l} = -\iint_A \frac{\partial \vec{B}}{\partial t}.d\vec{s}$$

$$\oint_C \vec{B}.d\vec{l} = \mu\iint_A (\vec{J} + \varepsilon\frac{\partial \vec{E}}{\partial t}).d\vec{s}$$

$$\oiint \vec{E}.d\vec{s} = \frac{1}{\varepsilon}\iiint_V \rho\ dV$$

$$\oiint \vec{B}.d\vec{s} = 0$$

Maxwell's Equations - Differential form

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu(\vec{J} + \varepsilon\frac{\partial \vec{E}}{\partial t})$$

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon}$$

$$\vec{\nabla}.\vec{B} = 0$$

Electromagnetic waves

$$\nabla^2\vec{E} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} + \mu\sigma\frac{\partial \vec{E}}{\partial t} + \frac{1}{\varepsilon}\vec{\nabla}\rho$$

$$\nabla^2\vec{B} = \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} + \mu\sigma\frac{\partial \vec{B}}{\partial t}$$

Energy density

$$u_E = \frac{1}{2}\varepsilon\left|\vec{E}\right|^2 \ , \ u_B = \frac{1}{2\mu}\left|\vec{B}\right|^2$$

Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

Intensity of radiation from a dipole

$$I = \frac{p_0^2\,\omega^4}{32\pi^2\,\varepsilon_0\,c^3}\frac{\sin^2\theta}{r^2}$$

Refractive index of a metal

$$n^2 = c^2\left(\mu\varepsilon - i\frac{\mu\sigma}{\omega}\right)$$

Phase velocity

$$v = -\frac{(\partial\varphi/\partial t)_x}{(\partial\varphi/\partial x)_t}$$

Superposition of waves $$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_1 - \alpha_2)$$

Fresnel Coefficients

$$r_{perp} = \left(\frac{E_{0r}}{E_{0i}}\right)_{perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{-\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$t_{perp} = \left(\frac{E_{0t}}{E_{0i}}\right)_{perp} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

$$r_{parl} = \left(\frac{E_{0r}}{E_{0i}}\right)_{parl} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$t_{parl} = \left(\frac{E_{0t}}{E_{0i}}\right)_{parl} = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

Elliptical polarization $$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\varepsilon = \sin^2\varepsilon$$

Young's slits (ideal case) $$I = 4I_0\cos^2\frac{\delta}{2} \quad , \quad \delta = ka\sin\theta$$

Thin film interference $$\Lambda = 2n_f d\cos\theta_t$$

Michelson Interfeometer $$m\lambda = 2d\cos\theta_m \quad , \quad \theta_p = \sqrt{\frac{p\lambda}{d}}$$

Multiple beam interference (thin film)

$$I_r = I_i \frac{F\sin^2\frac{\delta}{2}}{1 + F\sin^2\frac{\delta}{2}} \quad , \quad F = \left(\frac{2r}{1-r^2}\right)^2$$

$$I_t = I_i \frac{1}{1 + F\sin^2\frac{\delta}{2}}$$

Single slit diffraction $$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2 \quad , \quad \beta = \frac{kb}{2}\sin\theta$$

N harmonic oscillators (ideal case) $$I = I_0 \frac{\sin^2\frac{N\delta}{2}}{\sin^2\frac{\delta}{2}} \quad , \quad \delta = ka\sin\theta$$

Maxwell-Boltzmann distribution $$\frac{N_j}{N_i} = e^{-(E_j - E_i)/kT}$$

Blackbody energy density / Einstein coefficients $$u_\nu = \frac{8\pi h\nu^3}{c^2}\frac{1}{(e^{h\nu/kT} - 1)} = \frac{A_{ji}}{B_{ji}}\frac{1}{(e^{h\nu/kT} - 1)}$$