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GALWAY

SEMESTER 1 (WINTER) EXAMINATIONS 2000-2001

3rd Year B.Sc. Unit EP325 : Quantum Physics

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Time allowed : TWO hours.	Answer THREE questions

Note: A list of some formulae is given at the end of this paper.

Q.1 Explain what is meant by the term *blackbody*. Briefly suggest how a near perfect blackbody can be produced for experimental purposes.

Derive an expression for the number of allowed modes of electromagnetic waves in a cubic cavity of side length L. Hence write down the Rayleigh-Jeans formula for the energy density of a blackbody as a function of frequency.

Illustrate in the form of a rough graph, as to how the Rayleigh-Jeans distribution compares with the observed experimental distribution.
In relation to the blackbody spectrum explain what is meant by *Wien's Displacement Law*. Briefly mention how you might measure the surface temperature of the Sun.

Q.2 A particle of mass m is in a potential well defined by $V(x) = 0$ for $|x| \leq a$, $V(x) = \infty$ for $|x| > a$.

Solve the time independent Schrodinger Equation to find the allowed wavefunctions and particle energies.

Sketch the waveforms for the two lowest energy solutions.

Q.3 Use the Schwartz Inequality to derive the following form of the Heisenberg Uncertainty Relationship

$$\sigma_A^{\,2}\,\sigma_B^{\,2} \;\geq\; -\frac{1}{4}\left[\int \Psi^* \left[\hat{A},\hat{B}\right]\Psi\,dx\right]^2$$

Determine the commutator of the position and momentum operators. Hence show that the uncertainties in position and momentum are related by

$$\sigma_x\,\sigma_{p_x} \;\geq\; \frac{\hbar}{2}$$

Q.4 Discuss of the properties of weakly interacting identical particles which are; (i) classical particles, (ii) Bosons, (iii) Fermions.
In each case, construct an expression for $P(n_s)$, the probability of a particular macroscopic distribution, in terms of statistical weights of cells, g_s. (Note: you are not required to maximise these expressions.)

Q.5 Answer <u>two</u> of the following

(a) Show that non-degenerate energy eigen-functions are ortho-normal
The state function of a system can be expressed as a linear combination of energy eigen functions as follows

$$\Psi(x,t) \;=\; \sum_n c_n\,\psi_n(x)\,e^{-\frac{E_n}{\hbar}t}$$

Calculate the expectation value for energy $<E>$. Hence, comment briefly on the physical interpretation of c_n.

(b) Discuss the continuity properties required of physically acceptable wave functions. <u>Briefly outline</u> the method by which the Schrodinger equation can be solved for the case of a bound particle in a finite potential well.

(c) Derive an expression for the energy of the ground state electron wave function in a Hydrogen atom.

Schrodinger Equation: $i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x,t)\right]\Psi(x,t)$

Taylor's Series: $f(a+h) = f(a)+hf'(a)+\frac{h^2}{2!}f''(a)+\frac{h^3}{3!}f'''(a)+\ldots$

Binomial Expansion: $(x+y)^n = x^n+nx^{n-1}y+\frac{n(n-1)}{2!}x^{n-2}y^2+\ldots$

Normal Distribution: $f(x) = \frac{1}{\sigma\sqrt{2}\pi}\exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$

Standard Deviation: $\sigma = \sqrt{\frac{1}{N}\sum_i^N\left(x_i-\mu\right)^2}$

Schwartz Inequality: $\int f^*f\,dx\int g^*g\,dx \geq \frac{1}{4}\left[\int(f^*g+g^*f)\,dx\right]^2$

Stirling's Approximation: $\ln n! \approx n\ln n - n$

$$\nabla^2\Psi=\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2} = \frac{1}{r^2\,Sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\,Sin\theta\frac{\partial\Psi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(Sin\theta\frac{\partial\Psi}{\partial\theta}\right)+\frac{\partial}{\partial\phi}\left(\frac{1}{Sin\theta}\frac{\partial\Psi}{\partial\phi}\right)\right]$$

$$\frac{d}{dx}\ln f(x) = \frac{1}{f(x)}\frac{d}{dx}f(x)$$

$$\int u\,dv = uv-\int v\,du$$

$$\int_0^\infty x\,e^{-x}\,dx = 1$$

$$\int_0^\infty\frac{x^3\,dx}{e^x-1} = \frac{\pi^4}{15}$$

$$I_0 = \int_0^\infty e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$I_1 = \int_0^\infty x\,e^{-\alpha x^2}\,dx = \frac{1}{2\alpha}$$

$$I_n = \int_0^\infty x^n\,e^{-\alpha x^2}\,dx\,,\quad I_{n+2} = -\frac{dI_n}{d\alpha}$$

$$Sin(A+B) = Sin\,A\,Cos\,B+Cos\,A\,Sin\,B$$

$$Cos^2 A = \frac{1+Cos2A}{2}$$