GX512

Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

SUMMER EXAMINATIONS 2001

FINAL CIVIL ENGINEERING EXAMINATIONS

Structural Analysis II

Professor R. Falconer
Professor P. E. O'Donoghue
Dr. K. McNamara

Time allowed: **two** hours

Answer **3** Questions

1. (a) The stress function, ϕ = Cr$\theta sin\theta$ is a valid stress function for a line load P/unit length on the edge of a half space (C constant). Determine an expression for the constant C in terms of the load P and show that the radial stress is constant on the circumference of a circle that is tangent to the surface at the point of load application – see Fig. 1a. What are the other stress components on such a circle. (10 marks)
Recall:

$$\sigma_r = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}$$

$$\sigma_\theta = \frac{\partial^2 \phi}{\partial \theta^2}$$

$$\sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial r}\right)$$

(b) The thin rectangular plate in Fig. 1b is subjected to the in-plane loading as shown. The plate contains an elliptical cut-out where the dimensions of the ellipse are much less than that of the plate. If the aspect ratio of the ellipse is 7, determine the magnitude of the stress at point A – the tip of the major axis. (10 marks)

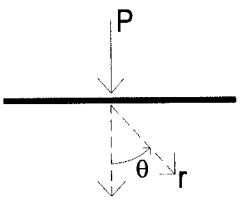

Fig 1a

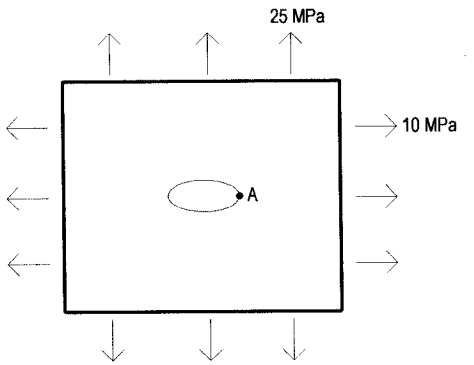

Fig 1b

2. (a) A space truss is frequently an assemblage of tetrahedrons; discuss. (6 marks)

(b) For the space truss in Fig. 2,

(i) Determine the members with zero internal force. Are any reaction forces zero. (5 marks)

(ii) Take moments about member 16.

Take moments about member 15.

Sum the force components in the y direction.

From these equations, evaluate the reaction forces for the plane truss (5 marks).

(iii) By considering equilibrium of node 3, determine the forces in the members that meet at that point. (4 marks)

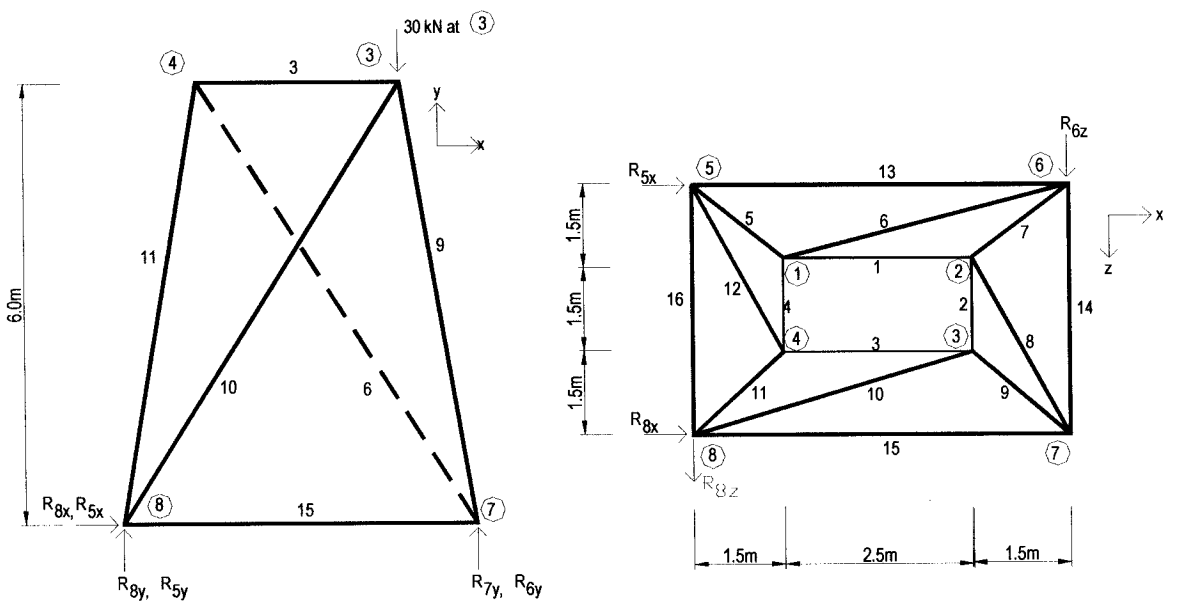

Fig 2

3. (a) Discuss the commonality between plate bending and grillage analysis. (6 marks)

(b) Determine the maximum deflection for a thin circular steel plate, radius ***a***, which is clamped all round and is subjected to a load of $q = q_o(a-r)$/unit area. What is the bending (fixing) moment at the clamped edge. (14 marks)
Recall:

$$\frac{d}{dr}\left\{\frac{1}{r}\frac{d}{dr}\left[r\frac{dw}{dr}\right]\right\} = \frac{Q}{D}$$

4. (a) Explain briefly the principal ideas of the Rayleigh-Ritz method as applied to finding the natural frequencies and mode shapes of beams. (8 marks)

(b) Using two shape functions, $\phi_1(x)$ and $\phi_2(x)$, in the Rayleigh-Ritz expansion, show that the method gives the following matrix relationship:

$$[K]\{C\} = \omega^2[M]\{C\}$$

where [K] is a 2x2 stiffness matrix, {C} is a vector of the constants C_1 & C_2, ω^2 represents the natural frequencies and [M] is a 2x2 mass matrix. (6 marks)

(c) For the simply supported beam shown inn Fig. 3, select appropriate shape functions for $\phi_1(x)$ and $\phi_2(x)$ and then write down expressions for each of the four terms in the stiffness matrix [K] and the mass matrix [M] given in part (b). (6 marks)
NOTE: Integrations need not be carried out.

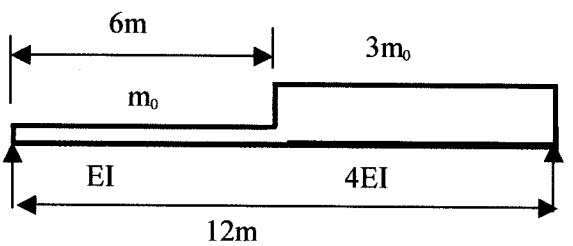

Fig. 3