GX577

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

Summer Examinations 2001

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B.Sc. (Honours) Degree Examination(4BS)

Mathematics Option:
*IE428 - '**Quality Control**'*

Dr. Wright,
Prof. O'Kelly,
Dr. Sheil.

Time allowed: ***Two*** hours

Attempt ***three*** questions

Cambridge Statistical Tables supplied.
Mathematical('log') Tables available

Q1.

How are CUSUM charts constructed? [2 marks]

If $S_r = \sum_{i=1}^{r}(x_i - m)$, show, using standard notation, that $E(S_r) = r(\mu - m)$ [2 marks]

What implications does this result have for

(i) the basic interpretation of CUSUM plots? [1 marks]

(ii) selection of the CUSUM reference value? [1 marks]

Describe two methods for the formal (statistical) analysis of CUSUMs [4 marks]

A company monitors one of its processes by analyzing samples of output at regular intervals. Sample means for the last 16 such samples are listed below:

8.00 8.01 8.02 8.01 8.00 8.01 8.06 8.07
8.05 8.04 8.03 8.05 8.06 8.04 8.05 8.06

(i) construct a CUSUM plot from this data. [5 marks]

(ii) Now provide detailed comment/opinion on the process performance over the period of time that this data was generated. [2 marks]

Q2.

(a) Differentiate between *p-charts*, *np-charts*, *c-charts* and *u-charts*, by stating in each case:

(i) the process (quality) parameter which the chart monitors/controls, [2 marks]

(ii) the sample/subgroup quantity/'statistic' which is plotted. [2 marks]

(b) What are *demerits* systems? [1 marks]

In complex products, many different types of defects occur. Such defects may be classified according to their importance/severity. Consider a classification system employing k such classes. Let $C_1, C_2, C_3, \ldots\ldots, C_k$ represent the numbers of defects from each class, detected on inspecting a sample of product.
Let $d = w_1C_1 + w_2C_2 + w_3C_3 + \ldots\ldots + w_kC_k$; where $w_1, w_2, \ldots$are 'weights' whose values reflect the relative seriousness of the different defect classes.

Derive expressions for a centreline and 3-sigma control limits for a control chart on d.
[Note: Assume that C_i , $i=1, 2, \ldots, k$ are independent $Poi(\lambda_i)$ random variables.] [4marks]

(c) Portable radios are subjected to final inspection. Defects noted are classified as 'functional' or 'appearance' defects. Recent results show that 18 functional and 150 appearance defects were recorded in inspecting a total of 500 radios.

(i) Assuming an inspection unit of 10 radios, compute centrelines and 3-sigma limits for separate *c-charts* for functional and appearance defects, respectively. [5 marks]

(ii) If functional, appearance defects are weighted 50 and 1 respectively, design a *d-chart* (using an inspection unit of 10 radios) to monitor future production. [3 marks]

Q3.

Consider using an $\bar{x}$-*chart* with (action) limits at $\mu_0 \pm 3.09\sigma_0/\sqrt{n}$.
If the process mean 'slips' from μ_0 to $\mu_0 + \theta$, whilst process variability remains fixed, show (employing standard notation and assumptions) that subsequent values will plot between the control limits, with probability;

$$P(\theta) = \Phi(3.09 - \theta\sqrt{n}/\sigma_0) - \Phi(-3.09 - \theta\sqrt{n}/\sigma_0)$$ [3 marks]

If t is the time lapse between samples, show that the ARL (represented as m_θ), expressed in units of time, is;

$$m_\theta = t/(1 - P(\theta))$$ [5 marks]

Using the fact that the equation above may be alternatively written as $t/m_\theta = 1 - P(\theta)$, construct a table of, and then plot, t/m_θ versus $\theta\sqrt{n}/\sigma_0$. [7 marks]

An $\bar{x}$-*chart* must be designed for a process which produces 10 items per minute, and for which $\sigma_0 = 1$. It is a design requirement that when $\theta = 2$, the ARL is 150 items. Use the sketch produced above to determine the value of t that should be employed when the sample/subgroup size is $n = 6$. [2 marks]

Q4.

Write a technical essay on **Process Capability Analysis**.
Your essay should explain what the term means and should list reasons for conducting such analyses. You could also consider the tools/techniques/measures used in conducting and reporting capability studies, how to interpret these, etc.
A minimum of 17 specific points, pieces of information, is envisaged and your presentation must take the form of an essay - a cryptic listing of points will attract minimal credit. [17 marks]

Q5.

A vendor ships components in lots of size 5000. Incoming inspection based on a single attributes sampling plan with n = 100, c = 2 is employed, in the customer's plant.

(i) Sketch the OC-curve for this plan (use the Poisson distribution/tables) [4 marks]

(ii) At what level of lot quality θ – the lot proportion nonconforming - will lots be rejected 90% of the time? [1 mark]

(iii) Find the plan, having c = 0, which will also reject with probability 0.9 those lots having the proportion nonconforming θ found in (ii) above. [2 marks]
Sketch the OC-curve for this plan. [4 marks]

(iv) Compare/contrast the OC-curves from (i) and (iii) and comment accordingly [1 marks]

(v) Suppose the vendor's process average is 0.5% nonconforming.
What is the probability of rejecting such lots
- under the n = 100, c = 2 plan ? [1 mark]
- under the plan with c = 0, derived in (iii) above ? [1 mark]

Calculate the average amount of inspection and the AOQ, at this lot quality, for both plans. [3 marks]
[*Note:* $I(\theta) = N - (N - n).P(\theta)$, $A(\theta) = \theta P(\theta)$]