OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY.

SUMMER EXAMINATIONS

2001

B.E. DEGREE EXAMINATION
INDUSTRIAL ENGINEERING AND INFORMATION SYSTEMS

# *ADVANCED OPERATIONS RESEARCH, RELIABILITY AND SAFETY ANALYSIS*

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**Time allowed:** 3 hours
**Answer :** Any 5 Questions

**Q 1** Given the following stochastic transition probability matrix, P, with transition rates in 'per hour' terms

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.85 & 0.05 & 0.10 \\ 0 & 0.90 & 0.10 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

**Note** : There are three states in the system, 1 - fully operational, 2 - derated but operational and 3 - failed.

i) Draw the state space diagram. (20% of marks)

ii) What is the MTTF given that the system started out in State 2? (30% of marks)

iii) What is the Reliability of the system at time equal to 2.0 hours, given

a) that the system started out in State 1

and

b) that the system started out in State 2? (40% of marks)

iv) Have you any explanation of the result obtained at iii a) and iii b) above?

(10% of marks)

**Q 2** i) A system has a failure density function (p.d.f. of life) given by

$f(t) = at \exp(-1/2\, at^2), t \geq 0,$ a = constant

At time t = 0, there are 6000 operating systems. After 5 hours, 4700 systems are still operating. What is the expected number of failures in the time interval of 15 to 20 hours on the assumption that each system has the above p.d.f. of life.

(50% of marks)

ii) Show that, in general, the failure rate (age specific failure rate; hazard function) of a series system is the sum of the failure rates of its components.

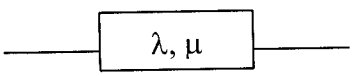
(15% of marks)

iii) A 3004 active redundant system consists of four identical exponential life units with failure rate $\lambda$. The units fail independently. Show that the MTTF of this system is given by

$$MTTF_{3004} = \frac{7}{12\lambda}$$

(15% of marks)

iv) Determine $P_0(t)$ and $P_1(t)$ of a component with constant failure rate $\lambda$ and constant repair rate $\mu$




$P_0(o) = 1$

where 'o' is the operating state and '1' is the failed state. (20% of marks)

**Q 3** A train carrying cars from a factory consists of 50 wagons and 2 locomotive engines. The train is capable of travelling (operating satisfactorily) if at least one of the two locomotives are operating and all the couplings are functioning satisfactorily. These are couplings between the wagons, between the locomotives and between wagons and locomotives.

With two locomotives load sharing the failure rate of each locomotive is $\lambda_1$. However, if one locomotive is in the failed state, the failure rate of the other locomotive increases to $\lambda_2$. Once the train is halted good components cannot fail. All the couplings have the same failure rate $\lambda_c$. The locomotives and the couplings can be repaired but repairs can only start once the system has failed. Once repairs start they are continued until everything has been repaired. These are two repair persons available – one for the locomotives with a repair rate of $\mu_l$ and the other for the couplings with a repair rate of $\mu_c$.

i) Draw the Markov Diagram (State Space) of this system, indicating the transition probabilities. Clearly indicate the down and the up states.

(20% of marks)

ii) Determine the steady state availability of the system i.e. the proportion of time the system is operating satisfactory if

$\lambda_1 = .1$ / hr ; $\lambda_2 = .2$ / hr

$\lambda_c = .01$ / hr

$\mu_l = 2$ / hr ; $\mu_c = 5$ / hr.

(30% of marks)

iii) A safety system consists of three monitors recording vital measurements of a process. The monitors are linked to a switching out device of a process. The process is maintained at regular intervals. The design objective of the safety system is to switch off the process if an unsafe situation arises between periods of maintenance.

Each of the three monitors may fail during periods between maintenance as follows:-

a) The monitor incorrectly indicates an unsafe situation (fail-safe) with probability 0.15.

b) The monitor does not detect an unsafe situation (fail-danger) with probability 0.05.

If the cost of an undetected unsafe situation of the process is 10,000,000 CU and the cost of switching off the system is 500,000 CU when there is no reason to do so, determine which of the following three possible operating strategies, A, B or C is optimal from a cost point of view:-

A) If at least one of the monitors indicate an unsafe state, the process is switched off.

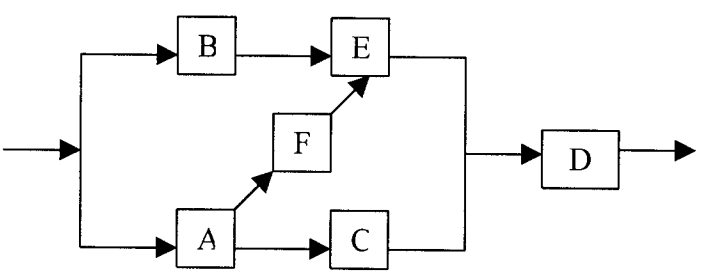
B) Only when all three monitors indicate an unsafe state, the process is switched off.

C) If at least two of the monitors indicate an unsafe state, the process is switched off.

Assume that the probability of an unsafe situation developing in the process during the period of interest is 0.001 and that the probability that the system, if in danger, would be tripped by the 'fail safe mechanism' may be neglected.

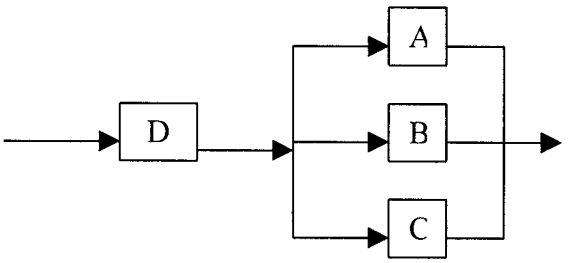
**Note:** CU = Currency Unit (50% of marks)

**Q 4** i) Determine (a) the reliability and (b) the MTTF of the following system:-




Assume each component is a constant failure rate component (exponential life) with parameter $\lambda$ = 0.02/annum.
[**Hint**: Conditional Probability Approach] (30% of marks)

ii) Determine the reliability of the system shown, if at least two of the three elements (A,B,C) must function properly for the system to function properly. Each element has a reliability of 0.90 (independent of time).




(10% of marks)

iii) Prove the memoryless characteristic of a constant failure rate (CFR) exponential distribution component.

(20% of marks)

iv) A CFR system with $\lambda = 0.0004$/hour has been operating for 2000 hours. What is the probability that it will fail in *a)* the next 100 hours? *b)* in the next 1000 hours?

(20% of marks)

v) What is the life expectancy of a CFR system with $\lambda = 0.0004$/hour at time $t = 1000$ hours given that the system has survived to 1000 hours.

(10% of marks)

vi) Show from fundamental principles that it is not possible to increase the MTTF of a CFR system by the use of fixed period scheduled maintenance.

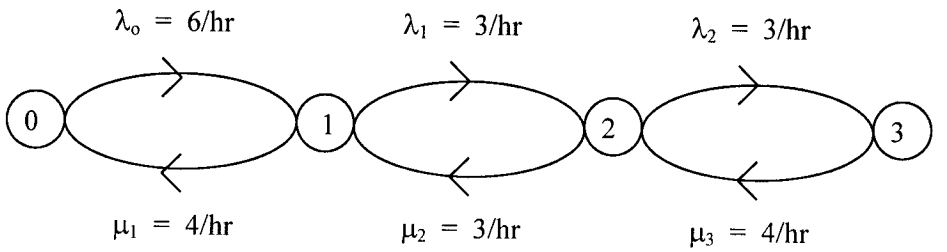
(10% of marks)

**Q 5** i) Give an outline proof of the following Pollaczek –Khintcline Formula for Lq, the expected number waiting for service in the M/G/1 : : GD/∞/∞ system, with arrivals according to a Poisson Distribution with parameter $\lambda$, and Service Time Distribution with mean E(t) and variance Var(t).

$$Lq = \frac{\lambda^2 (E^2(t) + Var(t))}{2(1 - \lambda E(t))}$$

(50% of marks)

ii) Determine $L_s$, the expected number in the system and $W_q$, the expected time waiting for service for the following queueing system:-




State i, i = 0, 1, 2, 3 gives the number of units in the system.

[**Hint** : 4 States]

(50% of marks)

**Q 6** i) The following three proposals for unloading trucks are under discussion:-

| ***Proposal Description*** | ***Service Rate*** | ***Capital Cost*** | ***Energy/Manpower Cost During Operation*** |
|---|---|---|---|
| A Manual | 6/hour | £36K | £40/hour |
| B Semi Automatic | 9/hour | £54K | £100/hour |
| C Automatic | 30/hour | £117K | £150/hour |

Each system may be modelled as an M/M/1 : : GD/∞/∞ system and trucks arrive into the system at the rate of 4/hour (Poisson Distribution).

Delay time for each truck is costed on the basis of £50 per hour for the total time spent in the system. The Energy and Manpower Costs are only incurred while the unloading system is actually being operated (i.e. being utilised). The company uses a straight line depreciation policy for project evaluation purposes based on the following formula:

$$\text{Depreciation Cost to be charged per hour} = \frac{\text{Capital Cost}}{9000}$$

[9000 = 2.5 years of 300 days and 12 hours/day]

Which proposal would you recommend? Carefully explain the methodology being used.

[ **Hint:** M/M/1 : : GD/∞/∞ System (Usual Notation)

$P_o = 1 - \rho$ $\qquad$ $L_s = \rho/1 - \rho$

$L_q = \frac{\rho^2}{1-\rho}$ $\qquad$ $L = \lambda W$ ]

(50% of marks)

ii) Show that in the M/M/R : : GD/K/K system (usual notation – machine interference problem):

$$\lambda eff = \mu(R - \bar{R}) = \lambda(K - L_s) = \mu eff$$

where $\bar{R} = \sum_{n=0}^{R} (R - n) P_n$ is the expected number of idle servers

$$\lambda_n = (K - n)\lambda, \quad 0 \le n \le K$$
$$= 0 \qquad \text{elsewhere}$$

$$\mu_n = n\mu, \quad 0 \le n \le R$$
$$= R\mu, \quad R \le n \le K$$

(50% of marks)

**Q 7** As Managing Director of Independent Power Producer Ltd. (IPP) you have to decide between two different generation plant configurations ($a_1$ or $a_2$) to generate electricity over the next fifteen years.

The payoff from each decision is given in the following matrix where $S_1$, $S_2$ and $S_3$ refer to levels of average growth in demand for electricity over the fifteen year time horizon.

***Payoff Matrix***

| | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|
| $a_1$ | £20M | £24M | £30M |
| $a_2$ | £ 6M | £13M | £25M |

i) Which decision would you make if you wished to use the following decision criterion in the context of decision making under uncertainty?

a) Minimax regret
b) Maximax

(15% of marks)

Following a brainstorming exercise with your executive colleagues you have decided on the following subjective (apriori) probabilities of each average growth in demand state:

$P(S_1) = 0.4$; $P(S_2) = 0.5$; $P(S_3) = 0.1$

ii) Which decision would you make under the criterion of maximization of expected value (MEV)?

(10% of marks)

Irish Power Consultants Ltd. (IPC) have offered to predict the future state of the Irish Electricity Market for you for a fee of £1M. The track record, on similar assignments in the past, of IPC is as follows:

$P(O_i/S_j)$

| ***IPC Report*** | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|
| $O_1$ | .7 | .2 | 0 |
| $O_2$ | .3 | .8 | 1.0 |

iii) Prepare a report for your Board clearly outlining the different options facing IPP by way of a decision tree and include appropriate recommendations. Would you recommend that IPP should retain IPC at a fee of £1M? What is the Expected Value of your overall set of recommendations?

(40% of marks)

iv) If the following utility values were assumed:

| Monetary Value | £6M | £10M | £12M | £13M | £20M | £24M | £25 | £30M |
|---|---|---|---|---|---|---|---|---|
| Utility | 0 | 10 | 18 | 23 | 50 | 80 | 84 | 100 |

What action would you recommend under the criterion of maximising expected utility without using the services of IPC.

(20% of marks)

What additional information on utility values of different monetary returns would you require to determine the expected utility if using the services of IPC?

(15% of marks)