

OLLSCOIL NA hÉIREANN
The National University of Ireland

National University of Ireland, Galway.

Michaelmas Examinations, 200001
BE Degree (Mechanical & Biomedical) Examination

Heat Transfer

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Attempt Four Questions

Time Allowed Three Hours

The following Tables are available:

Copies of Tables A.1 to A.3.2, A.7 to A.11, 3.1, 3.2, 12.3 & 12.4 from V. Wylen & Sonntag
Copies of Tables 6.2, 7.1 to 7.7, 8.4, 9.3, 12.1, A.4 to A.6 and A.8 and Figs. 7.10, 11.10 to 11.19, 12.15, 13.4 to 13.6 from Incropera & Dewitt. Stefan-Boltzmann Constant = $5.6697 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$

1. A hot-wire probe is a device for measuring fluid velocity. It consists of a fine wire (typical diameter $20 \mu\text{m}$) maintained at constant temperature by a control system which passes an electric current through the wire. The current determines the power dissipation in the wire, which in steady state, must equal the heat transfer from the wire. The heat transfer is largely convective, and therefore depends on fluid velocity. Hence, the velocity can be determined.
A platinum-coated tungsten wire is used at a temperature of 100°C as a hot-wire probe in air at 30°C . It is oriented normal to the direction of air flow, and it is required to measure velocities up to 100 m/s . In a calibration procedure, it is found that air velocity U_∞ is related to the wire current I by an equation of the form $U_\infty = (aI^2 + b)^2$, where a and b are constants.
 - (a) Show that the calibration equation above is approximately consistent with a standard empirical convection correlation. (10)
 - (b) Assuming that heat transfer from the wire is described accurately by the standard convection correlation, calculate the measurement error due to use of the approximate calibration equation when the true velocity is 100 m/s . (5)
 - (c) The arrangement described above suffers from the drawback that the freestream fluid temperature must be known. However, if two identical hot-wires are installed close to each other and operated at slightly different temperatures, air temperature can be determined as well as velocity. Derive a general equation for air temperature T_∞ in terms of the temperatures of the two hot-wires, the electric currents I_1 and I_2 in the wires, and other properties of the air and wires. Assume that air properties near the two hot-wires are similar. (10)

- 2(a) Consider a long body of uniform cross-sectional area A , which exchanges heat by convection at heat transfer coefficient h with an ambient fluid at temperature T_∞ . There is no convection on the end surfaces of the body – one end is maintained at a fixed base temperature T_b and the other is perfectly insulated. The cross-section perimeter length is P and the material conductivity is k .

Derive a differential equation for the variation of temperature along the body's length, and show that the solution to the equation is of the form:

$$T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}, \text{ where } m^2 = \frac{hP}{kA}.$$

Also derive expressions for the constants C_1 and C_2 in terms of material and geometric properties, and the base and ambient temperatures. (12)

- (b) A microprocessor chip of dimensions $A \times B \times C$, as shown in Figure 2, is cooled by convection with ambient air which is driven by a fan. Cooling is enhanced by an array of fins mounted perpendicular to one of the chip's large faces.

Derive an expression for the required length of the fins, as a function of fin thickness t , spacing d , height L , conductivity k and power \dot{Q} dissipated by the chip. Heat transfer through the side and bottom surfaces of the chip is small in comparison with heat transfer through the top surface and the fins. (9)

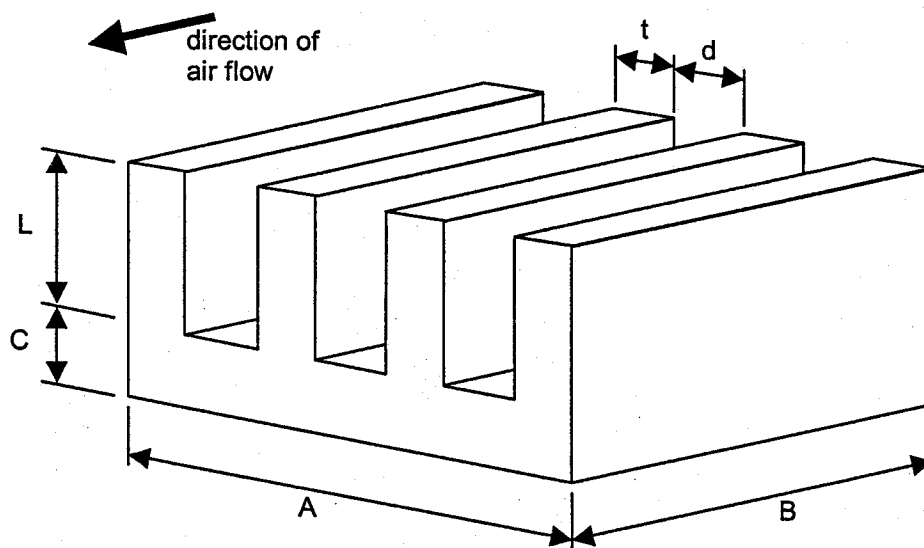


Figure (2) Microprocessor chip with fin array.

- (c) In a particular case of the configuration described in Figure (2), power dissipation is 25 W, the chip's dimensions are $A = B = 60$ mm and $C = 2$ mm, air temperature is 25°C and the convection coefficient is $22 \text{ Wm}^{-2}\text{K}^{-1}$. If the fins are to be 1 mm thick and 15 mm high, how many fins must be installed to keep the chip temperature at 60°C or less? (4)

3. The temperature of a small component in a combustion system is to be controlled by means of a cool airflow, which is fed through embedded tubes in the component. Air is delivered to these cooling tubes from a source located several metres away through additional tubing, but only 100 mm of each tube is embedded in the component. The delivery tubing is insulated and has the same diameter as the embedded tubes. At the operating condition of interest, the component temperature is uniform at 430 K, the inlet air temperature is 370K, and the air velocity in the tubes is 45 m/s. The properties of the air vary very little as it moves through the tubes. In the design of the cooling system, the following three possible configurations are examined:

- A. 8 tubes of diameter 6 mm
- B. 42 tubes of diameter 1.2 mm
- C. 100 tubes of diameter 0.5 mm

(a) For each case, calculate the overall heat transfer from the component to the air. (15)

(b) Based on the calculations for part (a), rank the three options in order of increasing Nusselt number. Also rank the options in order of increasing heat transfer rate. Explain both rankings, and any discrepancies between them, in terms of physical mechanisms. Use no more than 200 words. (10)

(Note this correlation for turbulent flow in a circular pipe with $10 < L/D < 400$:

$$Nu_D = 0.036 Re_D^{0.8} Pr^{1/3} (D/L)^{0.055}.$$

- 4(a) The spray drying process is widely used in the food and pharmaceutical industries to remove moisture from particles, or from liquid droplets which contain dissolved solids. The particles or drops are blown through hot dry air so that liquid is removed by evaporation.

A spherical droplet of diameter 0.45 mm is injected at a velocity of 50 m/s into the drying chamber of a spray drier, which contains dry air at 92 kPa and 80°C. The droplet is composed of water with 10% dissolved solids (by mass) and its initial temperature is 20°C. The contribution of dissolved solids to the volume of the droplet is small.

Calculate the values of the following at a time just after the droplet enters the spray drier:

- (a) the rate of mass transfer from the droplet; (6)
- (b) the net rate of heat transfer to the droplet; (7)
- (c) the rate of change of droplet diameter; (6)
- (d) the rate of change of the dissolved solids mass fraction (i.e. the ratio of the mass of dissolved solids to the total droplet mass). (6)

- 5 A wall is heated convectively, with a heat transfer coefficient of h_1 , by a fluid at temperature $T_{\infty 1}$. As shown in *Figure*, square internal channels carry coolant fluid through the wall. Heat transfer between the internal surfaces of the channels and the coolant (at temperature $T_{\infty 2}$) is characterised by the heat transfer coefficient h_2 . Also shown in *Figure* is a grid of finite difference nodes which are to be used to predict the temperature distribution in the wall. The wall is very long both in the horizontal direction (as drawn) and the direction normal to the plane of the diagram. Fluid conditions are uniform.

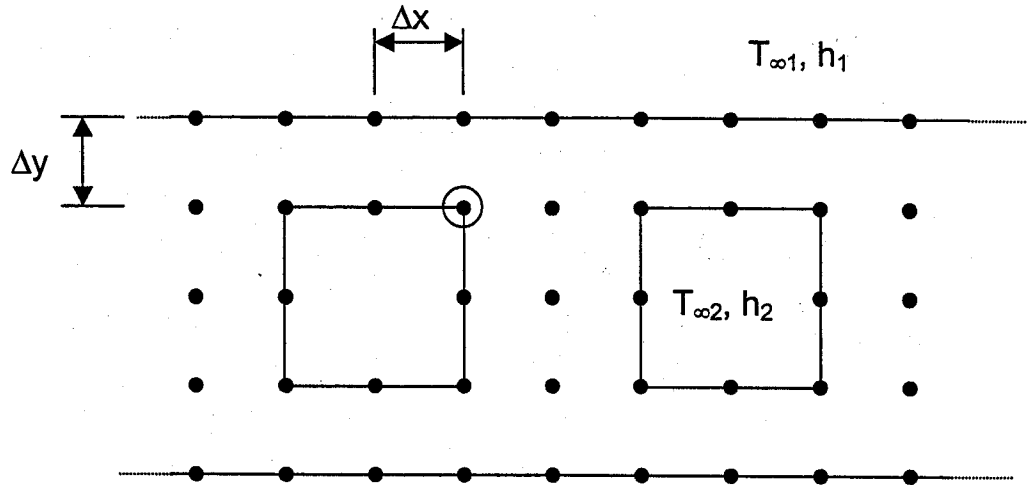


Figure 5 Wall with internal cooling channels, and finite difference grid.

- (b) Derive, in detail, a finite difference equation for a node at the corner of an internal channel (such as the node circled in Figure 5). (8)
- (c) Assemble a full set of finite difference equations to predict the temperature distribution in the wall. Take advantage of symmetry to reduce the size of the problem. Required numerical values are as follows:
- wall conductivity $k = 57.1 \text{ Wm}^{-2}\text{K}^{-1}$; $\Delta x = \Delta y = 0.1 \text{ m}$; $h_1 = 686 \text{ Wm}^{-1}\text{K}^{-1}$;
 $h_2 = 400 \text{ Wm}^{-1}\text{K}^{-1}$; $T_{\infty 1} = 400^\circ\text{C}$; $T_{\infty 2} = 30^\circ\text{C}$. (10)
- (d) Carry out the first iteration of a solution procedure for the equation set formulated in (b) above. (7)

6. In a racing car's radiator, a finned cross-flow heat exchanger is used to reject heat from the engine cooling water, as shown in **Figure 6**. The water is fed at 0.8 kg/s through a bank of tubes which run across an approximately square duct measuring $320 \times 320 \text{ mm}$. The tubes extend across the full width of the duct and are distributed evenly over its height. The duct is mounted to the side of the car, aligned parallel to the direction of motion. Forward motion of the car results in a flow of air through the duct over the tubes. The specification for the radiator requires that at a straight-line car speed of 200 km/hr , when the water enters the radiator at 140°C , it must be cooled to 50°C . Ambient air temperature is 30°C .

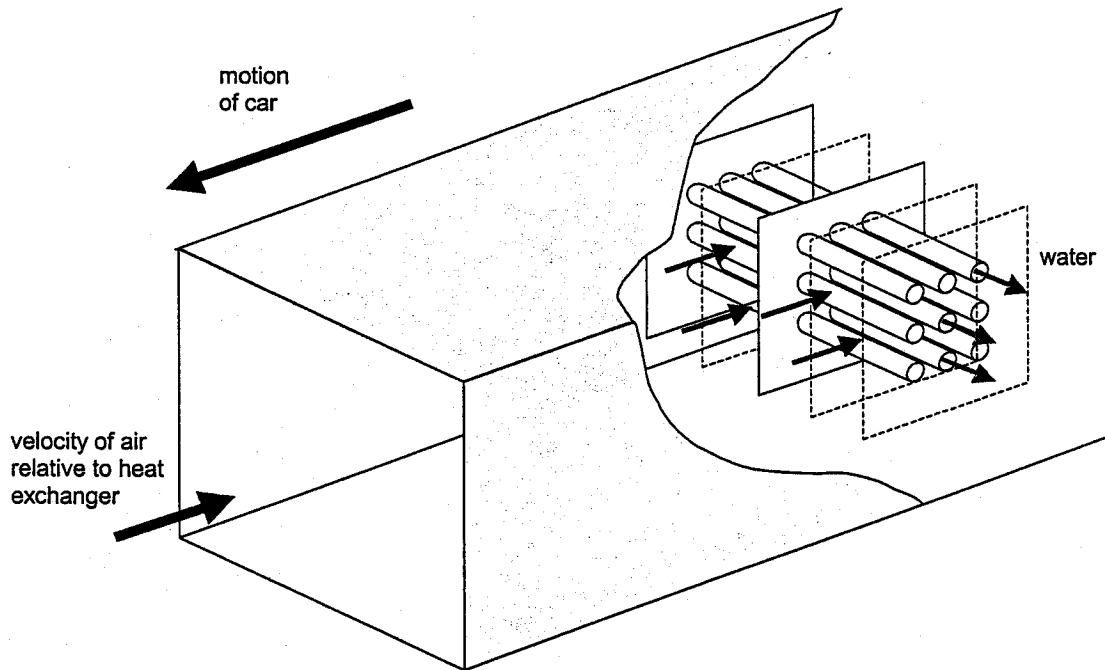


Figure 6 Schematic diagram of finned crossflow heat exchanger in car radiator. (For clarity only a few fins and tubes are shown).

- (a) What value of UA is required to achieve the specified performance, where U is the overall heat transfer coefficient and A is the heat transfer surface area? (12)
- (b) Suppose that the radiator is built with the UA value determined above. At a car speed of 80 km/hr , water enters the radiator at 100°C , and the overall heat transfer coefficient is 30% lower than at 200 km/hr . What is the water outlet temperature under these conditions? (13)

7. The spectral, directional emissivity of a diffuse surface at 2400 K varies with wavelength as shown in **Figure 7**.

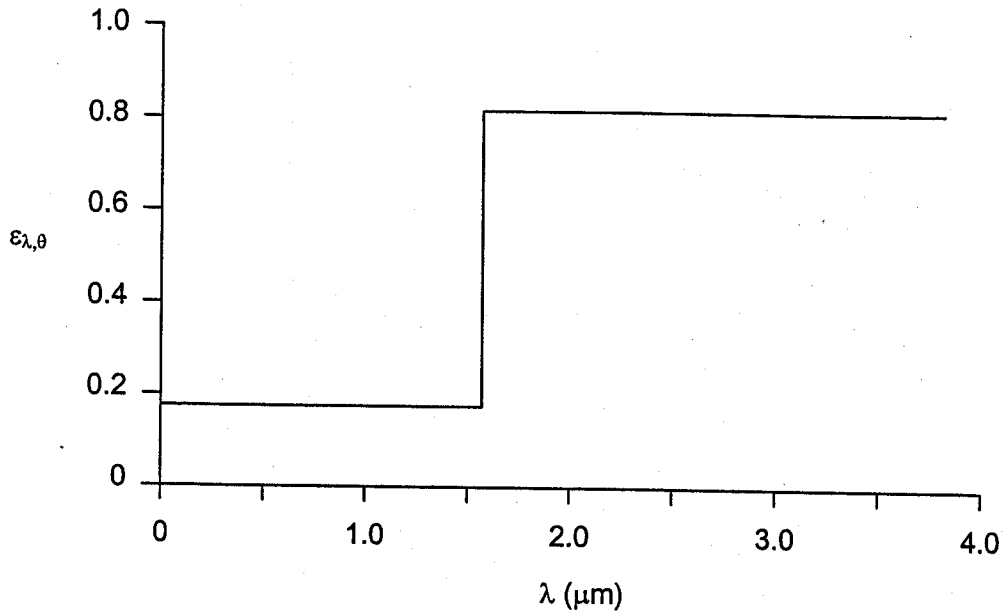


Figure 7 Spectral, directional emissivity of a diffuse surface at 2400 K

- (a) Determine the total, hemispherical emissivity of the surface at 2400 K. (12)
- (b) Determine the power per unit area emitted at wavelengths between 1 and 3 μm in directions within 10° of a line perpendicular to the surface. (13)