GX648

OLLSCOIL NA hÉIREANN
The National University of Ireland

National University of Ireland, Galway

Hilary Examinations 2000/2001

Third Year Mechanical and Biomedical Engineering Examination

FLUID DYNAMICS
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Attempt *Four* Questions
Time Allowed : 3 Hours

For every question attempted, produce at least one sketch or diagram which is clearly and accurately labeled with symbols and appropriate dimensions. State the assumptions your analyses are based upon.

The Equations Sheet, Moody Chart, Physical Properties Tables, Relative Roughness Chart and Vortex Shedding for Circular Cylinders Table are attached.

1. Air discharges from a large tank, in which the pressure is 700 kPa gauge and the temperature is 40° C, through a convergent nozzle of 25 mm tip diameter. Calculate the flow rates when the pressure outside the jet is (a) 200kPa, and (b) 550 kPa, and the barometric pressure is 101.3 kPa. Also, calculate the pressure, temperature, velocity and sonic velocity at the nozzle tip for these flow rates. [25]

   The following isentropic flow relations may be helpful:

$$\frac{V_2^2 - V_1^2}{2} = \frac{p_1}{\rho_1}\frac{\gamma}{\gamma - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right], \qquad \dot{m}_{max} = 0.6847\frac{A^* p_o}{\sqrt{RT_o}}$$

$$\dot{m} = A_2\sqrt{\frac{2\gamma}{\gamma-1}\, p_1 \rho_1\left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\gamma+1/\gamma}\right]}$$

2(a) The basic equations of flow may be non-dimensionalised to reveal several dimensionless parameters. Identify three of these parameters. Describe their constituents and the ratios they represent, and briefly outline the conditions under which they are important or negligible. [10]

(b) If the flow pattern is oscillating, with an inlet boundary condition of the form $u = U \cos \omega t$, what is the form, name and significance of the corresponding non-dimensional parameter? [5]

(c) Vortex shedding can be used to design a vortex flowmeter. A blunt rod stretched across a pipe sheds vortices whose frequency is read by a sensor downstream. Suppose the pipe diameter is 5 cm and the rod is a cylinder of diameter 8 mm. If the sensor reads 5400 counts/minute, estimate the volume flow rate of water in cubic metres per hour.

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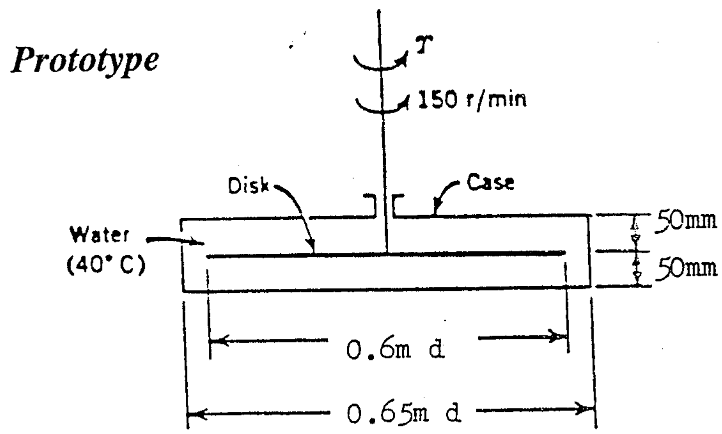
3 A circular disc of diameter d and of negligible thickness is rotated at constant angular speed ω in a cylindrical casing filled with a liquid of viscosity μ and density ρ. The casing has internal diameter D, and there is a uniform clearance y between both surfaces of the disc and the casing. Derive an expression for the torque required to maintain this speed if it depends only on the foregoing variables.

A model of the prototype rotating disc system, shown in ***Figure 3***, is to be tested in air at 20°C and 150 kPa (absolute). What dimensions and speed should the model have for the prototype and model torques to be identical? [25]

Figure 3



4. Water, $\rho = 998$ kg/m^3 and $\nu = 1.01 \times 10^{-6}$ m^2/s, is pumped between two reservoirs at 0.34 m^3/min through 120 m of 50 mm-diameter pipe and several minor losses (see ***Figure 4***). The roughness ratio is $\varepsilon/d = 0.001$. Compute the pump power required.

The following loss coefficients may be used:

K (sharp entrance) = 0.50 K (open globe valve) = 6.9
K (radiused bend) = 0.15 K (90° elbow) = 0.95
K (half-open gate valve) = 2.7

[25]

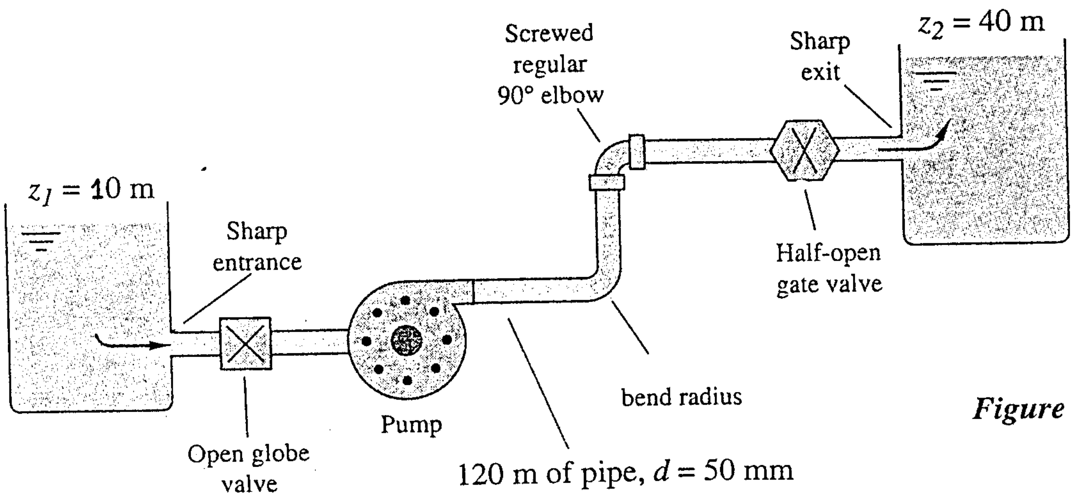


Figure 4

5 (a) The potential field of a combined vortex and sink can simulate the flow due to a stationary hurricane as shown in ***Figure 5a***, except that viscous effects can be neglected only outside a finite radius of about 35 m, as shown. Suppose that the pressure at $r = 35$ m is 1450 Pa less than the pressure far from the centre and the air density is 1.2 kg/m^3. Suppose further that the influx of air across the position $r = 35$ m is 3000 m^3/s per metre of depth into the paper.

Compute (i) the total inflow velocity V at $r = 35$ m; (ii) the sink strength $-m$ and vortex strength K in m^2/s; (iii) the pressure at $r = 100$ m compared with the pressure at infinity; and (iv) the angle β at which the streamlines cross the circle $r = 35$ m. [13]

(b) Air flows under steady, approximately one-dimensional conditions through a conical nozzle as shown in ***Figure 5b.*** If the speed of sound is approximately 330 m/s , what is the minimum nozzle-diameter ratio D_e/D_0 for which we can safely neglect compressibility effects if (i) $V_0 = 15$ m/s, (ii) $V_0 = 50$ m/s. [12]

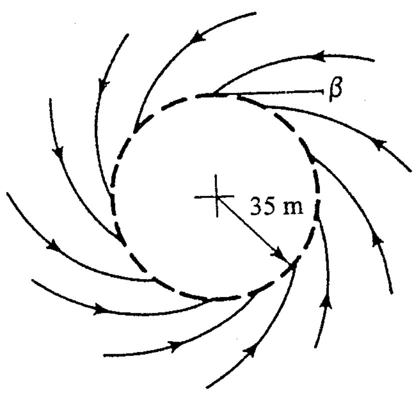


Figure 5a

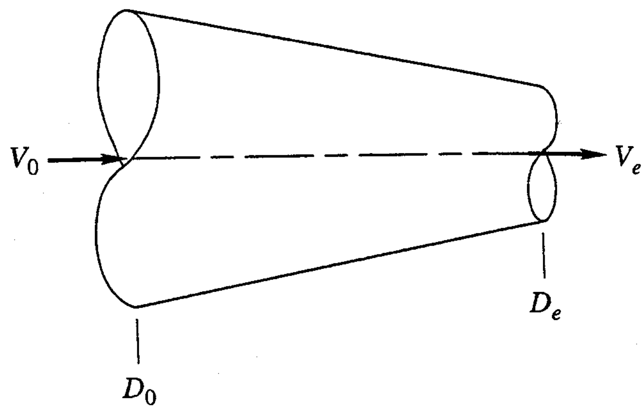


Figure 5b

6. The jet pump in ***Figure 6*** injects water at $U_1 = 40$ m/s through a central 75 mm-diameter pipe and entrains a secondary flow of water $U_2 = 3$ m/s in the annular region around the small pipe. The two flows become fully mixed downstream, where U_3 is approximately uniform. For steady incompressible flow, calculate U_3 in m/s.

The pressure $p_1 = p_2 = 175$ kPa, and the distance between sections 1 and 3 is 2000 mm. If the average wall shear stress between sections 1 and 3 is 330 Pa, estimate the pressure p_3. Why is it higher than p_1 ? [25]

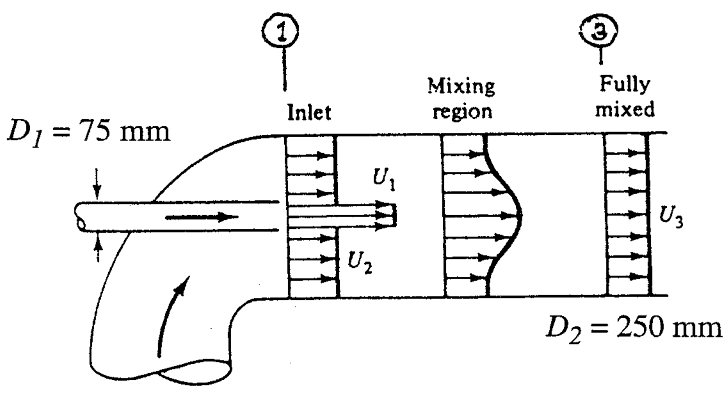


Figure 6

7 (a) Show that the drag force on a plate in a zero pressure-gradient viscous flow is given by the integral expression

$$D(x) = \rho b \int_0^{\delta(x)} u(U - u)\, dy$$

where b is the width of the plate, U is the freestream velocity, and x is the distance from the leading edge of the plate.
Derive expressions for the displacement and momentum thicknesses of the boundary layer.
Given the power-law approximations,

$$c_f = 0.02\, Re_\delta^{-1/6} \quad \text{and} \quad \left(\frac{u}{U}\right) = \left(\frac{y}{\delta}\right)^{1/7}$$

for the skin-friction coefficient and velocity profiles in turbulent flow along a smooth flat plate, prove that the boundary layer thickness δ is given by

$$\frac{\delta}{x} = \frac{0.16}{Re_x^{1/7}}$$

where

$$\frac{\tau_w(x)}{\frac{1}{2}\rho U^2} = 2\frac{d\theta}{dx} = c_f$$

and

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

[16]

(b) An underwater vehicle with a circular cross-section is 500 mm in diameter and 5 m long, and is propelled at 30 m/s in seawater (SG = 1.025). Estimate the power required to overcome friction drag if $Re_{crit} = 5 \times 10^5$.
[9]