

OLLSCOIL NA hEIREANN
The National University of Ireland

National University of Ireland, Galway

Hilary Examinations, 2000/01

Third Year Mechanical and Biomedical Engineering Examination

ANALYSIS OF VIBRATIONS & CONTROL SYSTEMS

Professor J.J. O' Connor

Professor J.F. McNamara

Professor P.J. Nolan

Attempt *Five* Questions, including at least two questions from each section.

Use separate answer books for each section.

Time Allowed: 3 Hrs.

SECTION A

The following are available : Laplace tables and semi-log graph paper

- 1(a)** Very briefly describe the Root Locus method and explain its use in the analysis of control systems. (10)
- (b)** A control system has the following open loop transfer function:

$$\frac{K}{s(s+1)(s+2)(s+3)}$$

Sketch the root locus for the system and determine the maximum value of the gain, K , for stability. (10)

- 2.** A control system has an open loop transfer function:

$$\frac{K}{s(s+6)}$$

- (a)** Determine the phase margin for the system if the gain (K) is 120. Determine also the damping ratio at the above gain. (8)
- (b)** State the transfer function for a phase lag controller, sketch its frequency response, and determine the phase lag controller parameters required to yield a phase margin of 65 degrees in the above system. (12)

- 3(a) Determine the transfer function relating output displacement y_1 , to input displacement x_1 for the hydraulic servo system shown in Figure 3. The constant relating the flow rate into the power cylinder to spool valve displacement is C_1 . Load reaction and inertial effects may be neglected.
- (b) Write down the transfer function for a three term controller (PID) and sketch the frequency response. (8)

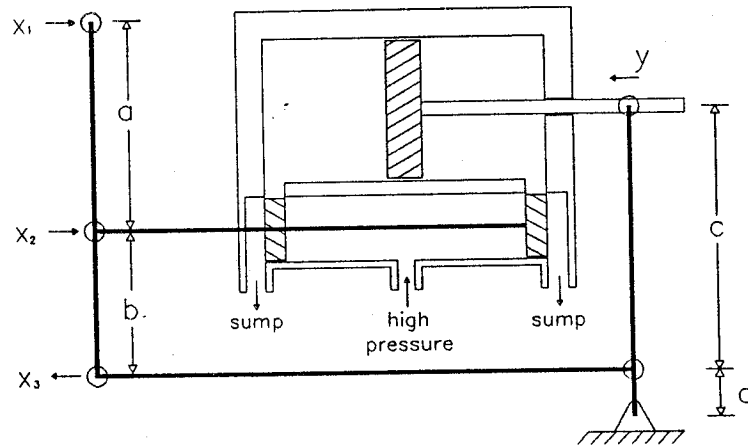


FIGURE 3

- 4(a) Describe Bode's asymptotic method for determining the frequency response of a control system. What approximations are used? What is meant by the term "corner frequency". Explain also how phase and gain margin can be read from the Bode plot. (10)
- (b) The graph shown in Figure 4 is a Bode magnitude response for a particular control system. Write down the transfer function and comment on the stability of the system. (10)

Bode Magnitude Plot

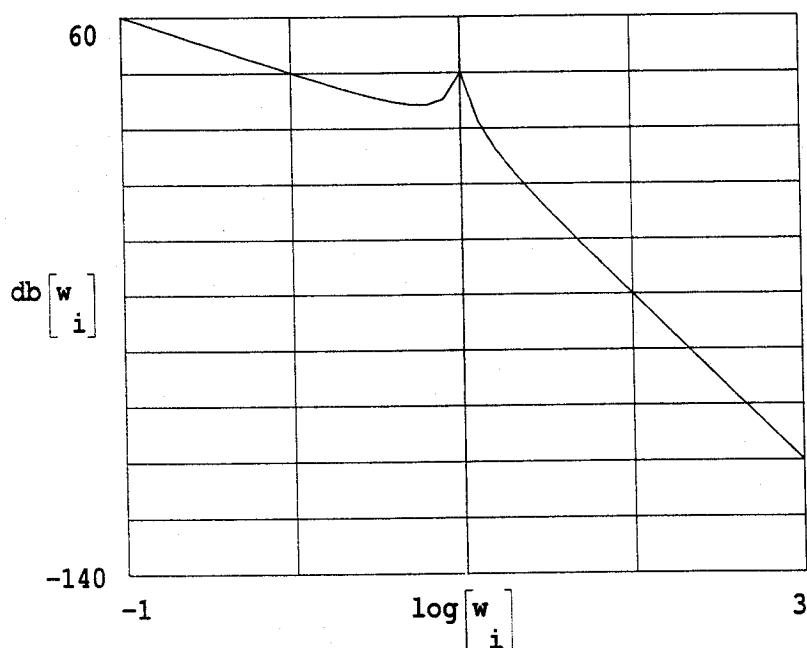


FIGURE 4

SECTION B

5. The system shown in Figure 4 is a simplified representation of a vehicle moving on a rough road. Displacement x is measured from the equilibrium position in the absence of input displacement. The stiffness and dashpot coefficient representing the suspension system are k and b , respectively.
- (a) Determine the transfer function relating output displacement (x) to input displacement (y). (8)
- (b) The surface of a road varies sinusoidally as shown. The amplitude and period are 0.05 m and 6 m respectively. If $m = 1200$ kg and $k = 400$ kN/m, determine the amplitude of the vehicle's displacement if the speed is 25 km/hr. Is there any particular speed which should be avoided? (12)

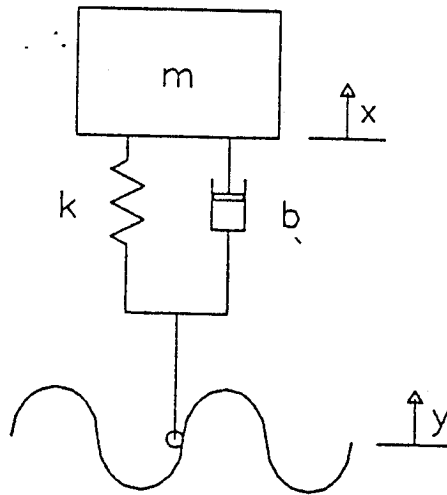
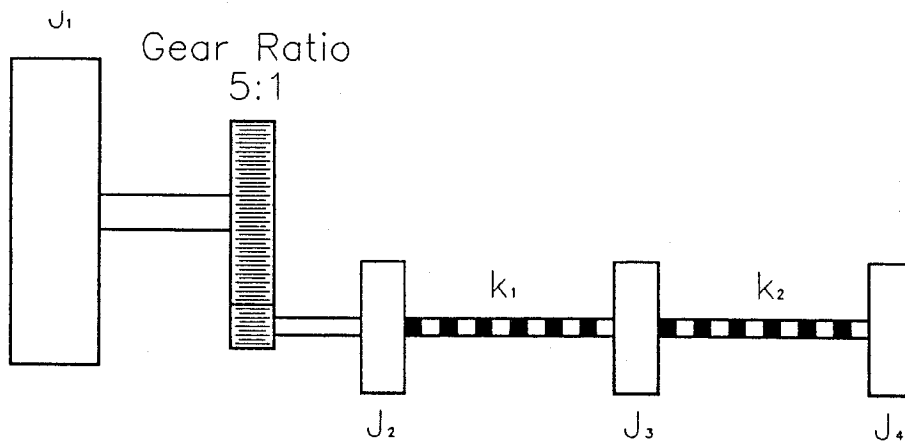


FIGURE 5

- 6(a) Using a simple two mass model explain the general principles describing the operation of a vibration absorber. Derive the expression for the frequency response of the original system (undamped) and that with the auxiliary mass (damped system) when an unbalanced force acts on the main mass. (10)
- (b) A machine which has a mass of 100 kg is observed to exhibit large amplitude oscillations at 10 Hz. Determine the parameters of a vibration absorber (size of auxiliary mass and spring constant) which will eliminate the vibration at 10 Hz and not introduce any resonances within $\pm 25\%$ of 10 Hz. (10)

7. An unrestrained rotational mechanical system comprising four rotating masses, a gearbox and two torsional springs is shown in Figure 7.

- (a) Write down the differential equations describing the motion of the rotating masses. (6)
- (b) Determine the natural frequencies and mode shapes (eigenvectors) for the system using an analytical approach. (6)
- (c) Using an iterative numerical approach (e.g. Holzer's Method) verify that the first non zero natural frequency determined above is correct. (6)



$J_1 : 125 \text{ Kg-m}^2$
 $J_2 : 3 \text{ Kg-m}^2$
 $J_3 : 6 \text{ Kg-m}^2$
 $J_4 : 4 \text{ Kg-m}^2$
 $K_1 : 4 \times 10^6 \text{ N-m/rad}$
 $K_2 : 2 \times 10^6 \text{ N-m/rad}$

FIGURE 7

8(a) A two degree of freedom model for a lathe and its support is depicted in Figure 8. Explain the different coordinate systems which can be used and the corresponding equations describing the vibration of the system. Include a brief discussion on the various types of coupling that are possible. Determine expressions for natural frequencies and mode shapes. (10)

A particular machine tool has a mass of $m = 1000 \text{ kg}$ and a mass moment of inertia of $J_0 = 300 \text{ kg-m}^2$. The equivalent stiffness of the supports are $k_1 = 3000 \text{ N/mm}$ and $k_2 = 2000 \text{ N/mm}$ and are located at $l_1 = 0.5 \text{ m}$ and $l_2 = 0.8 \text{ m}$.

Determine the natural frequencies and sketch the mode shapes. Suppose $k_1 l_1 = k_2 l_2$. What is the effect on the mode shapes? (10)

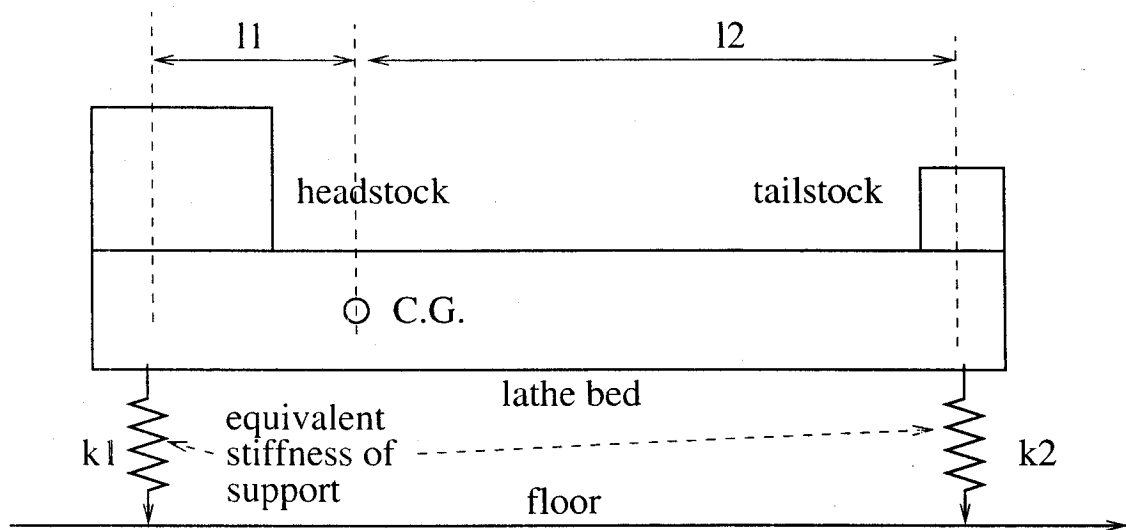


FIGURE 8