OLLSCOIL NA hÉIREANN
The National University of Ireland

National University of Ireland, Galway.

Trinity Examinations, 2000/01
B.E. Degree (Mechanical & Electronic) Examination

COMPUTER CONTROL OF ELECTRO-MECHANICAL SYSTEMS

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Attempt *Five* Questions
Time allowed: 3 Hours
Tables for Laplace, Z and W Transforms are provided.

1(a) Define what is meant by the z-transform of a sampled continuous signal, state the main theorems and briefly discuss the significance of z-transforms and z- transfer functions in the analysis and synthesis of digital control systems.

Discuss the various techniques for getting the inverse z-transform and for simulating the response of a digital system for an arbitrary input signal.

(10)

(b) Determine the response of a system with closed loop transfer function:

$$\frac{6+2.8\,z^{-1}-0.5\,z^{-2}}{1+0.7\,z^{-1}-0.25\,z^{-2}+0.175\,z^{-3}}$$

to a pulse input comprising a positive unit step at t=0 and negative unit step at the third sample.

(10)

2(a) Very briefly describe the phenomenon of transport lag in a control system. Provide examples of practical situations where it occurs and indicate its general effect on system stability.

(6)

(b) Given that the z-transfer function of a sampled continuous system with open-loop s-domain transfer function, G(s), is:

$$\frac{(z-1)}{z}\,Z\{L^{-1}\{G(s)/s\}$$

Determine the z-domain transfer function of the system if a pure transport-lag (time delay) is introduced in the system. The ideal sampler is followed by a zero-order-hold filter. *(6)*

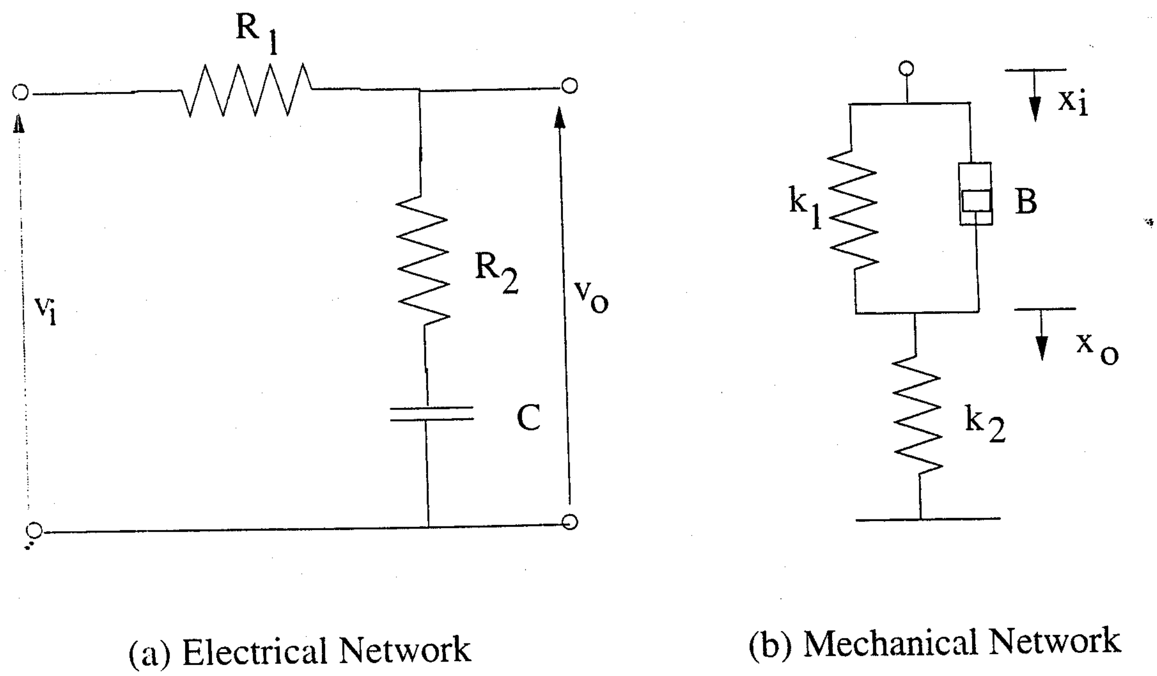
(c) A particular industrial process can be modelled as a Cohen and Coon system with a time lag of 1 sec and a pule time delay of 1.1 sec.

Determine the maximum gain for stability in the following cases:

(i) Analog proportional control is used.
(ii) Digital proportional control is used.

In the latter case, the sampling period is 0.2 sec. *(8)*

3(a) Determine the transfer function for the electrical and mechanical circuits depicted and plot the frequency response. Explain why they are called phase lag circuits and describe the steps used in tuning a phase lag controller to yield a specified phase margin in a control system. *(6)*



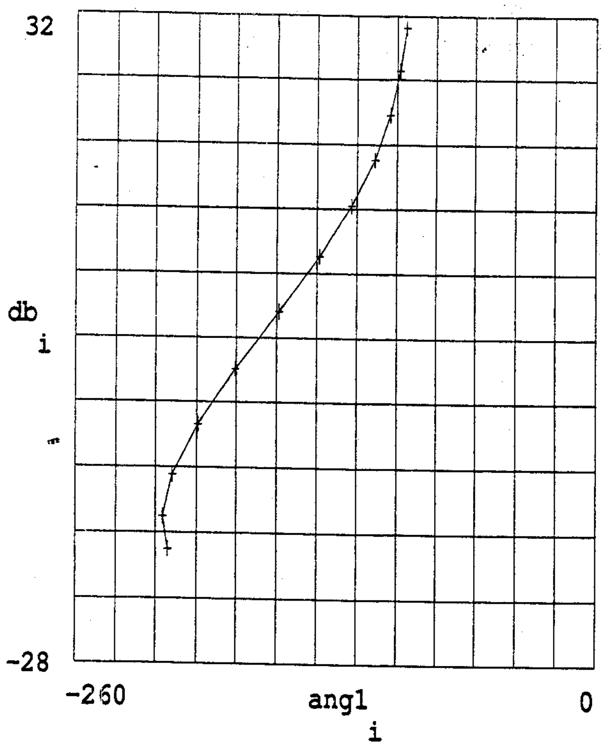
(a) Electrical Network (b) Mechanical Network

FIGURE 3(a)

(b) Very briefly describe the general rationale for the use of the W-transform in the analysis and synthesis of a digital control system. Are the results exact or is some inherent approximation used. *(6)*

(c) The Nichols plot and tabulated results for a particular control system are shown in Figure 3(c) Determine a z-domain controller transfer function which would yield a phase margin of 40 degrees.

(8)



w_i	db_i	ang_i
0.01	58.84	-90.2
0.016	54.84	-90.3
0.025	50.84	-90.5
0.04	46.84	-90.9
0.063	42.84	-91.4
0.1	38.83	-92.1
0.159	34.83	-93.4
0.251	30.82	-95.4
0.398	26.8	-98.5
0.631	22.74	-103.5
1.001	18.6	-111.1
1.586	14.27	-122.6
2.514	9.6	-138.7
3.985	4.48	-159
6.316	-0.92	179.4
10.01	-6.1	160.5
15.866	-10.68	147.9
25.148	-14.53	143.3
39.859	-17.52	145.9

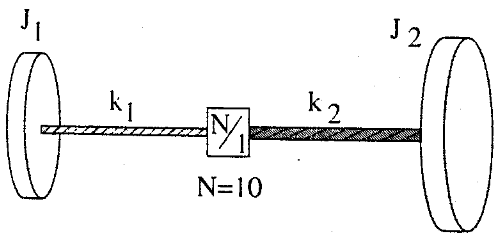
FIGURE 3(c)

4(a) Describe the use of discrete equivalents for analog controllers and filters in digital control systems. Describe some of the main methods used and cite some of their advantages and disadvantages.

(8)

(b) The drive train of a servomechanism is depicted in Figure 4. Determine the natural frequency. If a design constraint is that there must not be any interaction between the servo loop and mechanical torsional vibrations, write down a filter transfer function which could be used to eliminate the shaft resonance signal from speed feedback. Using, the Bilinear transformation or any other appropriate method, write down the z-domain transfer function of the filter.

(12)



$J_1 = 0.108$ Kg–m^2 $\quad J_2 = 14.2$ Kg–m^2

$k_1 = 5000$ N–m/rad $\quad k_2 = 500 \times 10$^3 N–m/rad

FIGURE (4)

5(a) Derive the general formula for the z-transfer function of a sampled continuous time system. An ideal sampler and zero-order hold filter can be assumed. You may also assume that system does not include any inherent time delays (transport lags). Show also, how such a sampled system can be approximately modelled as a continuous time system with an appropriate time delay. *(8)*

(b) A simplified block diagram model for a RPC (remote position control system)is shown in Figure 5. Apply the results derived above to determine the transfer function and using root locus or otherwise determine the maximum gain for stability. What is the corresponding value of gain if the above approximation us used. *(12)*

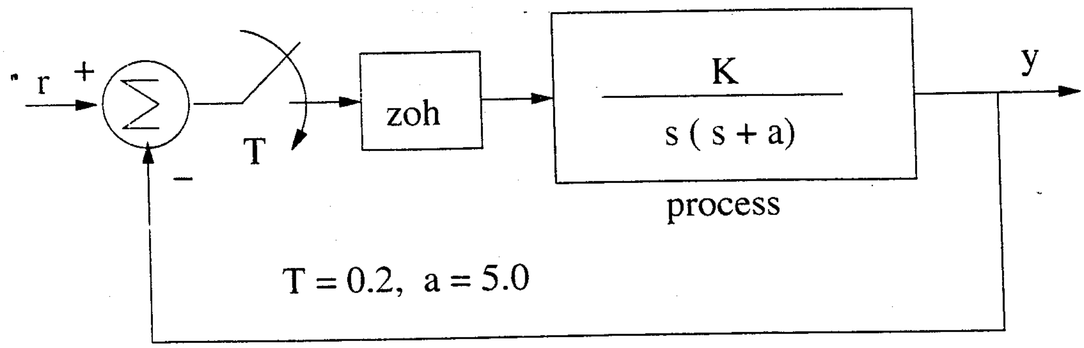


FIGURE 5

6. The diagram show in Figure 6 depicts an industrial oven which is digitally controlled. The control and disturbance inputs are reference temperature and environmental temperature, respectively. The output is over temperature. Proportional control is used and, as shown, a zero-order-hold filter is included.

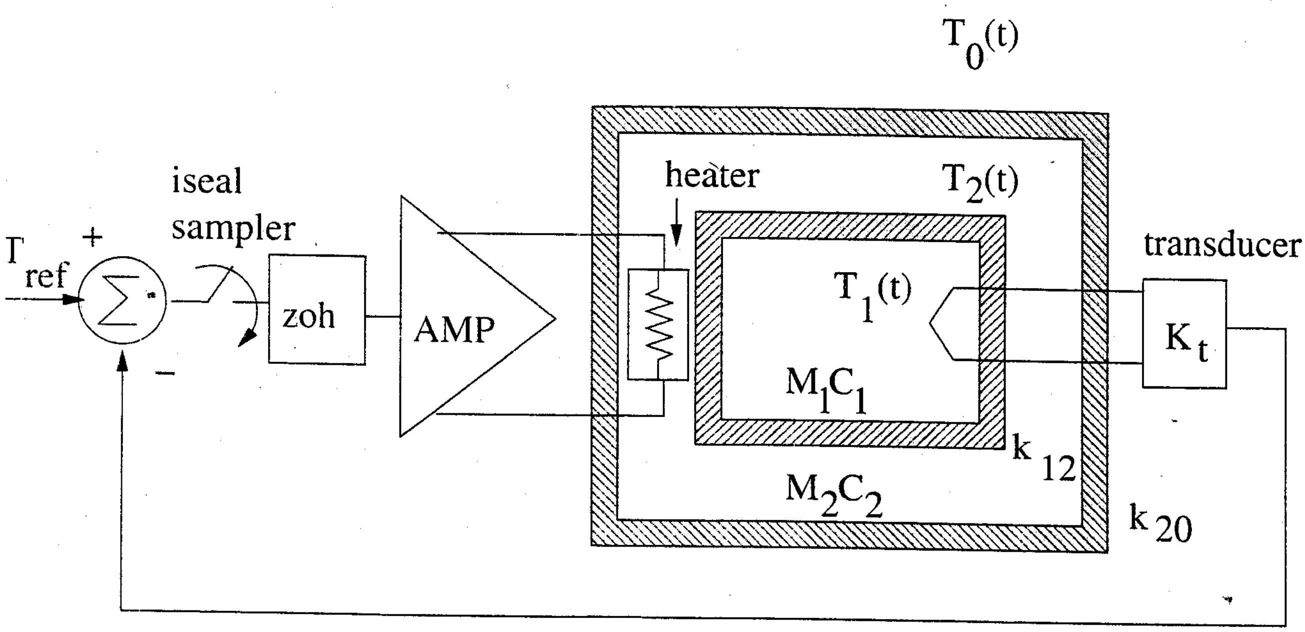


FIGURE 6

(a) Derive the transfer function relating oven temperature T_1 to reference temperature, $T_{ref.}$ *(6)*

(b) Write down an expression for the transfer function relating oven temperature to reference in the sampled system. It is not necessary to simplify the result. Explain also what assumption could be made so that a similar result would obtain for the disturbance input. *(6)*

(c) Determine, using an appropriate s-domain approximation for the system with sampler and zero-order-hold, an estimate for the gain at marginal stability. *(8)*

7(a) State the sampling theorem and briefly discuss its practical significance. *(6)*

(b) Derive the transfer function for the state variable filter shown in Figure 7. In the band-pass filter case, sketch the frequency response. Write down the transfer function for a discrete equivalent. *(8)*

(c) Sketch the frequency response of the analog and digital filter for the case of an appropriate and too low sampling rate. *(6)*

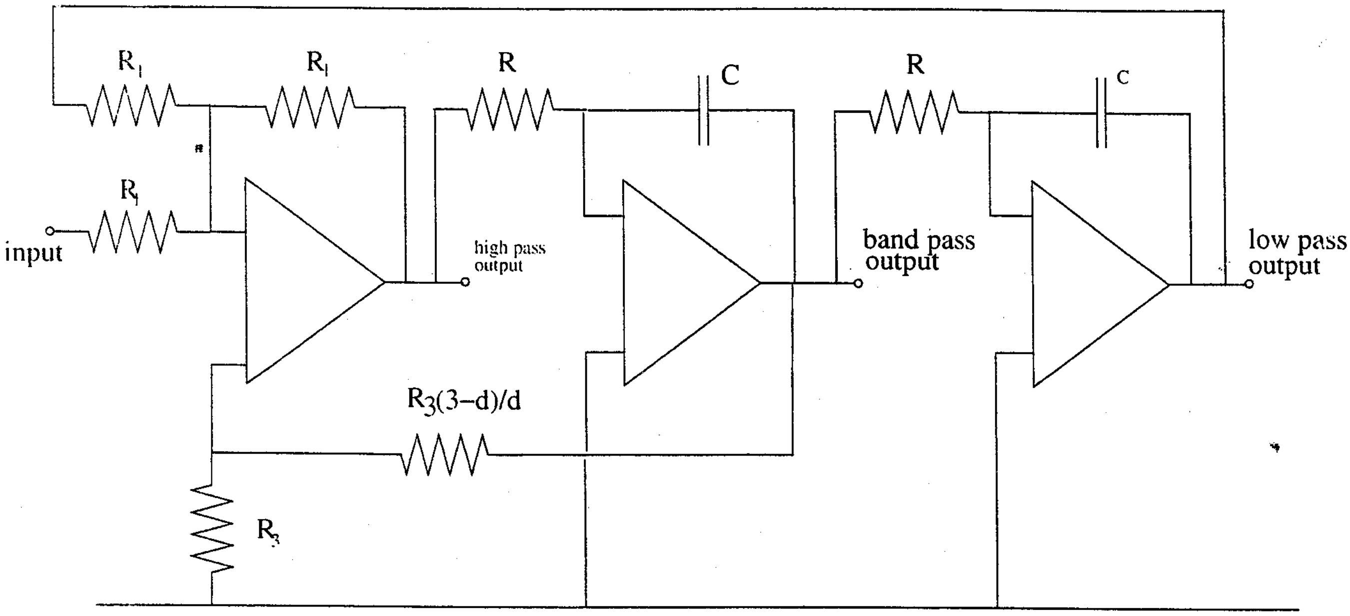


FIGURE 7

8(a) Discuss the general condition for stability of a digital control system in terms of the z-domain poles. Show the correlation between z-domain pole location and time response. Also show the mapping between (i) constant damped frequency, (ii) constant damping coefficient and (iii) constant damping ratio in the s-domain and their equivalent locus in the z-domain. *(12)*

(b) A particular control system which is sampled at the rate of 10 Hz. has the following open-loop z-domain transfer function:

$$\frac{0.025\,K(z-0.823)}{(z-0.950)\,(z-0.415)}$$

Determine:

(i) Maximum gain for stability and the frequency of the sustained oscillations at the stability limit.
(ii) Gain for a damping coefficient of 0.5 *(8)*