OLLSCOIL NA hÉIREANN, GAILLIMH

THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

WINTER EXAMINATIONS, 2000

THIRD YEAR CIVIL & ENVIRONMENTAL ENGINEERING EXAMINATION

ENGINEERING HYDRAULICS I (EH 306)

Examiners: Professor P.E. O'Connell
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Attempt *any* **five** questions: (Time: 3 hours)

(The Handout of Tables and diagrams provided may be useful in one or more questions)

1. **[Critical Depth Meter (Broad-crested weir)]**

(a) What *(very briefly!)* do you understand by each of the following terms:

'critical' depth	**Froude number** (Fr)
a 'broad-crested' weir	**the minimum 'specific energy' head** (e_{min})
'velocity of approach'	**weir submergence factor** (σ)
a 'mild-slope' channel	**a 'control' section**

[2 marks]

(b) In the context of a **steady discharge in an open channel of rectangular-section**, show that the **minimum specific energy head** satisfies the relation $e_{min} = \frac{3}{2} y_{cr}$, where y_{cr} is the **'critical' depth.** *[4 marks]*

(c) Recognising that, at the downstream end of the crest of **a broad-crested weir** which spans the *entire* width of a *rectangular* channel, **the flow will be 'critical'** (in order to *maximise* the discharge), **derive a theoretical expression** of the form

$$Q_{theoretical} = 1.705bH^{\frac{3}{2}} \text{ (metric)}$$

for the discharge over the weir, where H is the upstream head above the sill of the weir and b is the channel width, making the usual assumption of straight horizontal streamlines at the critical section and taking the Coriolis coefficient α=1.0. *[5 marks]*

(d) **A broad-crested weir** spans the *entire* 3 m width of a mild-slope **rectangular channel.** The sill height is P_s = 0.75 m above the bed of the channel and the water levels *just* upstream and downstream of the weir are h_1 = 0.5 m and h_2 = 0.1 m respectively above the level of the sill, so that the **submergence factor** is $\sigma = h_2/h_1 = 0.2$. **Check if**

(i) *neglecting* the **'velocity of approach'**, the **discharge** Q_1=1.808 m^3s^{-1}; *[3 marks]*

(ii) *considering* the **'velocity of approach'**, the **discharge** Q_2 = 1.878m^3s^{-1}. *[4 marks]*

(e) Explain *briefly* how the above equation for the broad-crested weir is adapted for a different **critical depth meter** in which the **critical depth is produced by a symmetric contraction of the channel** (as in a **Venturi flume**) rather than by a hump across the bed as in the case of the broad-crested weir. *[2 marks]*

2. **[Hydraulic Jump]**

(a) What *(very briefly!)* do you understand by *each* of the following terms:

a 'hydraulic jump', **'alternate' depths**, **'conjugate/sequent' depths**, and **'rapidly-varied' flow.** *[2 marks]*

(b) Describe *very briefly* the **physical conditions** which gives rise, in an open prismatic channel, to the formation of the **standing wave** known as a **'hydraulic jump'** and indicate *clearly* whether such a jump is a case of **gradually-varied flow** or **rapidly-varied flow.**

[2 marks]

(c) *Outline* the derivation of ***either*** of the following **conjugate/sequent depth relations for a hydraulic jump** in a *rectangular* open channel;

$$(Fr_1)^2 = \frac{v_1^2}{gy_1} = \frac{1}{2}\frac{y_2}{y_1}\left(1+\frac{y_2}{y_1}\right), \quad \textit{giving} \quad \frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8(Fr_1)^2}-1\right]$$

$$(Fr_2)^2 = \frac{v_2^2}{gy_2} = \frac{1}{2}\frac{y_1}{y_2}\left(1+\frac{y_1}{y_2}\right), \quad \textit{giving} \quad \frac{y_1}{y_2} = \frac{1}{2}\left[\sqrt{1+8(Fr_2)^2}-1\right]$$

where $y_1 < y_2$ are the upstream and downstream depths respectively, Fr and v being the corresponding Froude Numbers and average velocities respectively. *[5 marks]*

(d) **Supercritical** flow issues from under a **vertical sluice gate** spanning the entire width of a **rectangular channel,** the vertical height of the sluice gate opening section being 1.0 m. The *vena contracta* of the flow jet is located approximately 1.0 m downstream of the gate, the coefficient of contraction at that section being $C_c = 0.60$. The channel has width B = 4.0 m, bed slope $S_o = 0.005$ and a Manning roughness coefficient of n = 0.015.

For a **steady discharge** $Q = 20 m^3 s^{-1}$ in this channel,

(i) check if the **normal depth** $y_n = 1.2595$m and the **critical depth** $y_{cr} = 1.3659$m *[4 marks]*

(ii) hence show that the channel is of **'steep slope'.** *[1 mark]*

(iii) Suppose that your colleague used the **Direct Step Method** (DSM) to **plot the surface profile** downstream from the vena contracta, using depth increments of $\delta y = 0.04$m, up to the depth of 1.20m (i.e. twice the depth at the vena contracta), as given in the table below, and that the downstream depth is known to be regulated to 1.812m, (i.e. that the downstream surface profile is of constant depth = 1.812m), **show that this scenario gives rise to the formation of a hydraulic jump** between the section of the vena contracta and the downstream region of constant depth.

[2 marks]

Depth (m)	0.60	0.64	0.68	0.72	0.76	0.80	0.84	0.88
Distance(m)	0.0	11.125	22.361	33.715	45.199	56.830	68.632	80.639
Depth (m)	0.92	0.96	1.00	1.04	1.08	1.12	1.16	1.20
Distance(m)	92.899	105.478	118.470	132.021	146.363	161.903	179.456	201.057

(iv) **Using the given profile, estimate the conjugate depths of the jump** and **locate the starting point of the hydraulic jump,** relative to the location of the **sluice gate.**

[3 marks]

(v) Explain *clearly* why, in the above case, the **length of the jump is not a relevant factor** in obtaining your results for part (iv) above.

[1 mark]

3. **[Chezy/Manning-type Equations and Surface Profiles]**

(a) Show that, for **steady one-dimensional flow**, the dimensionless form of the **dynamic equation**, (obtained by dividing the acceleration $a = \dfrac{dV}{dt}$ by the gravity acceleration g) yields the **total energy slope** S_T equal to the **friction slope** $S_f = \dfrac{\tau_o}{\rho g m}$, which result, when combined with the **dimensional analysis** result for the variables involved, i.e.

$$\frac{\tau_o}{\frac{1}{2}\rho V^2} = \Phi\left(\frac{\rho V D}{\mu}, \frac{k}{D}\right) = \frac{f}{4}, \text{ giving } \tau_o = \frac{\rho f V^2}{8},$$

yields the **'Chezy-type' equation** $V = \sqrt{\dfrac{8g}{f}}\sqrt{mS_f} = C\sqrt{mS_f}$

and the **'Manning-type' formula** $V = \dfrac{1}{n} m^{\frac{2}{3}} S_f^{\frac{1}{2}}$, on substituting $C = \dfrac{m^{\frac{1}{6}}}{n}$. *[7 marks]*

(b) Show also how, under the conditions of **both steady and uniform flow**, the above Manning-type formula reduces to the standard **Manning formula for open channel flow.** *[3 marks]*

(c) Outline *very briefly* the role of the above **Manning-type formula** in the plotting of **steady non-uniform (but gradually-varied) surface profiles.** *[2 marks]*

(d) *List* some **applications of steady surface profiles** for solving open channel problems. *[2 marks]*

(e) Describe *briefly*, with the aid of sketches, how **surface profiles are used** to transform a *known* **Depth-Discharge relation** at a *given* section to that of another section a *known* distance upstream. *[6 marks]*

4. <u>Part A</u> (Answer <u>*either*</u> **Part A** below <u>*or*</u> **Part B**, *not both*)

[Forms of Surface Profiles]

(a) In the context of **steady gradually-varied non-uniform surface profiles in open channels,**
either explain *briefly* the basis of the following inequalities.

$$\left\{S_o - S_f\right\} = \frac{de}{dx} \lessgtr 0 \qquad \textit{according as} \qquad y \lessgtr y_n$$

$$\left\{1-(Fr)^2\right\} = \frac{de}{dy} \lessgtr 0 \qquad \textit{according as} \qquad y \lessgtr y_{cr}$$

or, using the **Manning formula**, show that for a **'wide rectangular'** channel, the **surface profile slope equation** reduces to

$$\frac{dy}{dx} = S_o\left[\frac{1-\left(\dfrac{y_n}{y}\right)^{\frac{10}{3}}}{1-\left(\dfrac{y_{cr}}{y}\right)^{3}}\right]$$

[10 marks]

Continuation of Q4. Part A:

(b) For the case of a **'wide rectangular' channel of 'mild' slope, either** (i) demonstrate how the above inequalities can be used with the **surface profile slope** equation to determine not only the **sign of the slope of the profile** at a section where the relative magnitudes of the actual depth y, the normal depth y_n and the critical depth y_{cr} are known but also the **general form of the profile** both upstream and downstream of such a section, **or** (ii) demonstrate how the same results can be obtained using the above **special form of the surface profile slope equation.** *[7 marks]*

(c) Hence, **sketch the three surface profiles** for the **'mild' slope case.** *[3 marks]*

4. Part B (Answer *either* **Part A** above *or* part **Part B** below, *not both.*)

[Surface Profile Calculations]

(a) In the context of **steady non-uniform (but gradually varied) surface profiles in open channels,** outline *very briefly* the basis of the different **simple finite difference schemes for plotting such surface profiles** both upstream and downstream of a section of known depth y, corresponding to the following **three dimensionless forms of the dynamic equation.**

$$\frac{de_s}{dx} = S_o - S_f \,; \quad \frac{dH_T}{dx} = \frac{d(e_s + z)}{dx} = -\,S_f$$

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \dfrac{\alpha Q^2 b}{g a^3}} = \frac{S_o - S_f}{1 - (Fr)^2} = \frac{\dfrac{de_s}{dx}}{\dfrac{de_s}{dy}}$$

where the *constant* discharge Q, the channel bed slope S_o, the Manning roughness coefficient n and *either* the form of the channel section (if the channel is prismatic) *or* plotted channel sections at known distances from the section of given depth (*or* stage) are *all given.* *[6 marks]*

(b) Water flows along a **"wide rectangular"** channel and discharges freely as **a waterfall at the end of the channel, giving 'critical' depth at that section.** The longitudinal bed-slope is $S_0 = 0.0005$ and the Manning $n = 0.025$. If the depth close to the edge of the waterfall (i.e. at the 'critical' depth section) is $y_{cr} = 0.1\text{m}$, **check** the following computer output solution;

(i) the **discharge per unit width** of $q = 99.04544$ litres per second. *[3 marks]*

(ii) the **'normal' depth** $y_n = 0.2670383\text{m}$. *[2 marks]*

(iii) the channel has a **'mild' slope.** *[1 mark]*

(c) **Sketch** the form of **longitudinal water surface profile** upstream of the waterfall, in relation to the *given* normal and critical depths. *[2 marks]*

(d) If the depth, at a given section (1) located upstream of the waterfall described in part (b), is $y_1 = 0.2\text{m}$, use *either* the **Numerical Integration Method** *or* the **Direct Step Method** to estimate the distance further upstream (or downstream ?) of that section for the depth to reach the value of $y_2 = 0.22\text{m}$.
(Use a **single step,** with $\delta y = 0.02\text{m}$).

[6 marks]

5. **[The 'Discharge Problem']**

(a) In the context of **open channel flow**, what (*very briefly*), you understand by *each* of the following terms:

the steady **'discharge' problem,** **'normal' depth** (y_n), **'critical' depth** (y_{cr}), **'uniform' flow,** **'steady' flow,** and **'specific energy' head** (e_s). *[2 marks]*

(b) A *long* **prismatic open channel leads from a large lake,** the **steady surface level** of which is at a height H above the bed of the channel at the lake outlet. Which of the following **two controls, 'critical' depth** or **'normal' depth,** occurs at the channel inlet and thereby determines the discharge if the channel is

(i) **mild slope;** (ii) **steady slope?** *[4 marks]*

(c) In part (b), what is the **significance of the term "long"** in relation to the channel? *[2 marks]*

(d) Suppose that, in part (b) above, $H = 3.5$ m and the **channel is of rectangular section** of width $B = 4$m, with Manning's $n = 0.025$ and bed-slope $S_o = 0.005$. Suppose further that two of your colleagues, **one assuming 'steep' slope and the other assuming 'mild' slope,** presented you with the following sets of **computer output results for obtaining the discharge Q** in the channel.

Slope Assumption	y_{cr} (m)	y_n (m)	Discharge Q (m^3s^{-1})	S_{cr}
Steep	2.33333	3.38767	44.65393	0.01296
Mild	2.09655	2.98174	38.03238	0.01246

Demonstrate **which set** (if *either*) **gives a sensible estimate of the discharge Q.** *[12 marks]*

6. **[Hagen-Poiseuille Equation for Laminar Flow]**

(a) *Outline* the **derivation of the Hagen-Poiseuille head-loss equation** and the corresponding **pipe resistance equation** for **laminar flow** of a **Newtonian fluid.** *[7 marks]*

(b) Crude oil, of density $\rho = 860$ kg m^{-3} and kinematic viscosity $\eta = (\mu/\rho) = 18.6 \times 10^{-6}$ m^2 s^{-1}, flows *down* from one open tank into another by means of a **straight vertical pipe** of length $L = 10$ m, diameter $D = 0.01$ m and absolute roughness $k = 0.03 \times 10^{-3}$ m. Each end of the pipe is "bell-mouthed" and a valve ($K = 0.19$) is fitted at mid-length. The bottom of the pipe is *well below* the oil surface level in the lower tank. Assuming that the pipe flows full and that the entry (but not the exit) loss is negligible, determine

(i) the **steady inflow** to the upper tank required to maintain a **steady oil surface level difference of H** = 12 m in the tanks. *[4 marks]*

(ii) the ratio of **form** (i.e. shock) losses to **friction** losses in the pipe. *[3 marks]*

(iii) the vertical **friction drag force F_d** exerted on the inside wall of the pipe. *[4 marks]*

(c) If, instead of the pipe described in part (b) above discharging into the lower tank under its liquid level, **it discharges freely into the atmosphere** at a point located $H = 12$ m below the liquid level in the tank supplying the flow Q, all the pipe characteristics remaining the same, **will this Q value be greater than, equal to, or less than that obtained for part (b) (i) above? Briefly explain your answer!** *[2 marks]*

7. **[Linear Matrix method for Ring Mains]**

(a) List all the **standard basic relations** which must be satisfied for the correct analysis of liquid flow in a **ring main pipe network.** *[3 marks]*

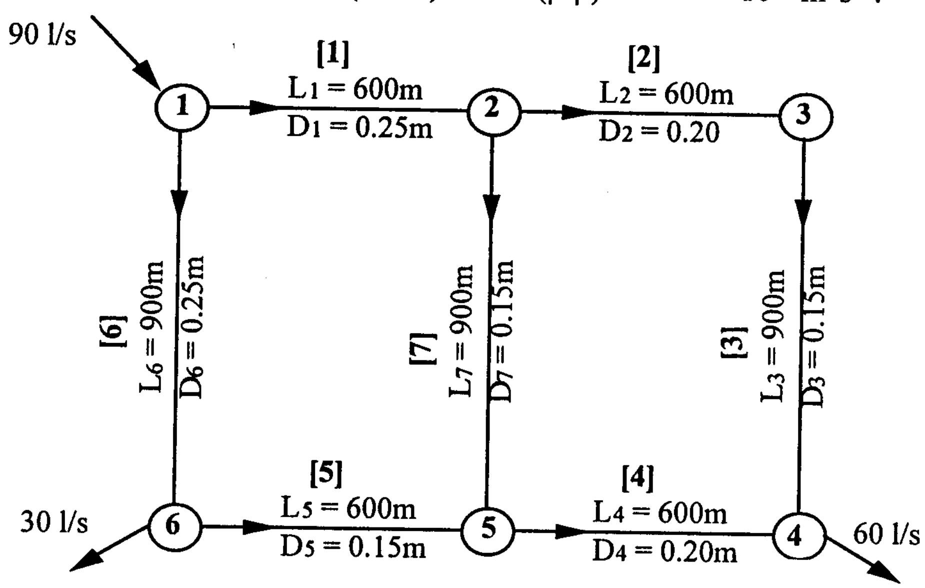
(b) Describe *briefly* the **'Linear Matrix' Method** for a ring main network and indicate the advantages claimed for the method vis-á vis the **Hardy-Cross finite difference method.** *[6 marks]*

(c) For the **ring main** sketched in the accompanying figure, the data are as follows:

Pipe No.	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Length(m)	600	600	900	600	600	900	900
Diameter(m)	0.25	0.20	0.15	0.20	0.15	0.25	0.15
Node Nos.	(1,2)	(2,3)	(3,4)	(4,5)	(5,6)	(1,6)	(2,5)

The **external flow rates** are $+0.09\,m^3s^{-1}$ (inflow) at node No.1, $-0.06\,m^3s^{-1}$ (outflow) at node No.4 and $-0.03\,m^3s^{-1}$ (outflow) at node No.6.

The absolute boundary roughness for each pipe is $k = 0{\cdot}03 \times 10^{-3}$ m and the kinematic viscosity of the fluid (water) is $\nu = (\mu/\rho) = 1.14 \times 10^{-6}\,m^2s^{-1}$.



Layout of Ring Main, showing the external inflow and outflows, the pipe lengths and the pipe diameters

Suppose that your colleague presented you with the following solution for the discharges, etc., in the ring main, **obtained using the matrix method**, based on the assumption that each i^{th} pipe has **form (shock) losses** amounting to $h_{i,\,form} = 2{\cdot}5\,(V_i^2/2g)$

$$
\begin{array}{l}
\text{Junction 1}\\
\text{Junction 2}\\
\text{Junction 3}\\
\text{Junction 4}\\
\text{Junction 5}\\
\text{Loop No. 1}\\
\text{Loop No. 2}
\end{array}
\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & -1 & 0\\
1 & -1 & 0 & 0 & 0 & 0 & -1\\
0 & 1 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 1 & 0 & 1\\
36.8293 & 64.8961 & 389.9368 & -102.4221 & -234.3513 & -63.9049 & 0\\
36.8293 & 0 & 0 & 0 & -234.3513 & -63.9049 & 334.5923
\end{bmatrix}
\begin{bmatrix}
0.040647\\ 0.022140\\ 0.022140\\ 0.037860\\ 0.019353\\ 0.049353\\ 0.018507
\end{bmatrix}
=
\begin{bmatrix}
-0.90\\ 0\\ 0\\ 0.06\\ 0\\ 0\\ 0
\end{bmatrix}
$$

Loop No. 1 consisting of **pipes** [1],[2],[3],[4],[5] and [6] and **Loop No. 2** consisting of **pipes** [1],[7],[5] and [6].

Continuation of Q7.

Unchecked Results of the Ring Main Analysis

Pipe	Discharge (m^3s^{-1})	Head Loss (m)	Friction Coeff. (f)	Reynolds No. (Re)
Loop No. 1:				
1	0.40647E-01	0.14970E+01	0.16807E-01	0.18159E+06
2	0.22140E-01	0.14368E+01	0.18087E-01	0.12364E+06
3	0.22140E-01	0.86332E+01	0.17568E-01	0.16485E+06
4	- 0.37860E-01	- 0.38777E+01	0.16628E-01	0.21142E+06
5	- 0.19353E-01	- 0.45354E+01	0.17923E-01	0.14410E+06
6	- 0.49353E-01	- 0.31539E+01	0.16309E-01	0.22049E+06
Loop No. 2:				
1	0.40647E-01	0.14970E+01	0.16807E-01	0.18159E+06
7	0.18507E-01	0.61923E+01	0.18046E-01	0.13780E+06
5	- 0.19353E-01	- 0.45354E+01	0.17923E-01	0.14410E+06
6	- 0.49353E-01	- 0.31539E+01	0.16309E-01	0.22049E+06

Using the converged Matrix Equation solution and the given table of 'Unchecked Results', spot check the given solution for each of the **standard basic relations** referred to in (a) above, paying *special attention* to the results for pipe No. [7]. *[8 marks]*

(d) Calculate the **ratio of form head loss to friction head loss** in Pipe No. [7]? *[3 marks]*

8. **[Pump-Pipeline System Analysis]**

(a) In the context of **pump-pipeline system analysis and pump selection,** what (*very briefly!*) do you understand by *each* of the following terms;

manometric head-discharge curve, efficiency-discharge curve and **system curve** ? *[2 marks]*

(b) **A variable-speed pump,** having the **manometric head-discharge relation** of the form $H_p = AQ^2 + BQ + C$, (for discharge Q in *litres per second* and H_P in *metres),* for which $A = - 0.0017$, $B = - 0.4500$ and $C = 60.3289$ is installed in a pumping station for the purpose of delivering sewage to a settling tank through a 0.2 m diameter μPVC pipeline, 2.5 km in length, the static head lift involved being $H_s = 15.0$ m.

The *effective* absolute roughness of the pipe is $k = 0.15 \times 10^{-3}$ m, the kinematic viscosity of the sewage is taken as $\nu = (\mu / \rho) = 1.14 \times 10^{-6}$ m^2s^{-1} and an allowance is made for form (i.e. shock) losses amounting to $(\Sigma K)V^2/2g = 10.0\ V^2/2g$.

Table A: Data for the pipe system curve, for a static head lift H_S = 15.00m.

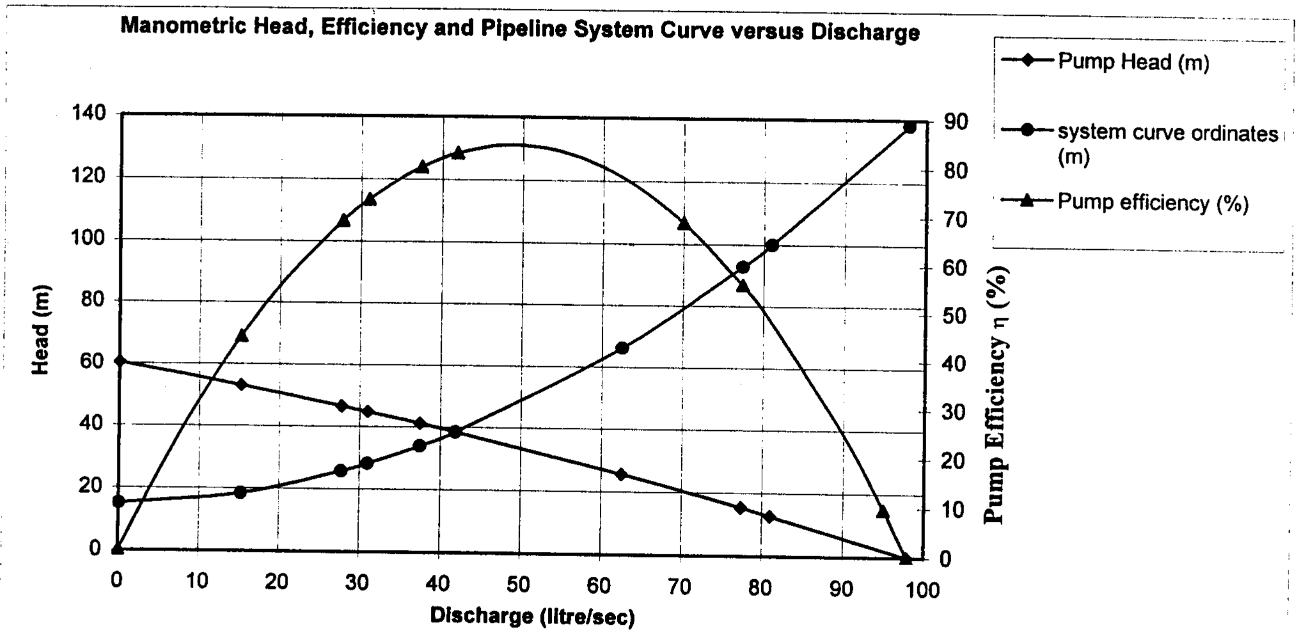
Discharge (litre/sec)	Reynolds no. (Re)	Friction factor (f)	Head loss (m)		Static head (m)	Pump manometric head (m)
			friction	shock		
0.05	279	0.03353	0.0001	0.0000	15.00	15.0001
15.25	85128	0.02184	3.2767	0.1200	15.00	18.3967
27.75	154904	0.02058	10.2215	0.3974	15.00	25.6189
31.00	173059	0.02040	12.6437	0.4960	15.00	28.1397
37.46	209107	0.02011	18.2032	0.7241	15.00	33.9272
41.89	233836	0.01996	22.5898	0.9055	15.00	38.4953
62.50	348884	0.01951	49.1652	2.0156	15.00	66.1809
77.40	432057	0.01932	74.6700	3.0912	15.00	92.7613
81.00	452153	0.01929	81.6237	3.3855	15.00	100.0092
97.87	546352	0.01915	118.3225	4.9431	15.00	138.2656

Table B : The Pump Characteristics and the Pipeline System Ordinates:

Discharge (litre/sec)	0.05	15.25	27.75	31.00	37.46	41.89	62.50	77.40	81.00	97.87
Pump Manometric Head (m)	60.3064	53.0711	46.5323	44.7439	41.0864	38.4953	25.5633	15.3146	12.7252	0.0000
System Curve Ordinates (m)	15.0002	18.3967	25.6189	28.1397	33.9272	38.4953	66.1809	92.7613	100.0092	138.2656
Pump Efficiency η (%)	0.17	44.35	68.50	72.98	79.66	82.55	77.83	55.78	48.11	0.00

The **pipe system curve data, and the pump characteristics** (manometric-head/discharge and efficiency/discharge data) for the **calibration pump speed** N_1 = 1450 rpm, are provided in the previous Tables A and B.

These data are also shown plotted in the accompanying graph.



Note that a separate version of the above figure is provided with this Examination Paper for submission with your Examination Answer Book.

Using the **enlarged version of the above Figure** provided separately with this examination question paper, **determine graphically the discharge** Q in the pipe **and the pump head** H_P delivered to the pipeline for the following **two** cases:

(i) **the pump is operated at the original calibration speed** $N_1 = 1450$ rpm. *[3 marks]*

(ii) **the pump is operated at the lower speed** $N_2 = 1200$ rpm. *[8 marks]*

(For case (ii), it is not necessary to plot the pump head discharge ($H_{p,2} - Q_2$) curve over the whole range of Q_2).

(c) By interpolation of the tables provided (*if necessary*), **check analytically your solution for the discharge Q** in the pipe for *either* of the above two cases. *[7 marks]*

(d) **Estimate the power consumption** (in watts) **of the pump** for the original pump calibration speed $N_1 = 1450$ rpm. *[2 marks]*

Note again that a separate version of the figure for Question No. 8 is provided with this Examination Paper for submission with your Examination Answer Book.