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NATIONAL UNIVERSITY OF IRELAND
GALWAY

SEMESTER II (SUMMER) EXAMINATIONS 2001

B.Sc. (Honours) Experimental Physics: Paper II

EP417: Solid State Physics

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Time allowed: TWO hours.	Answer THREE questions.

Q.1 A monochromatic beam of X-rays of wavevector **k** can be scattered elastically by a regular lattice of scattering centres into a beam with wavevector **k'** where

$$\mathbf{k'} = \mathbf{k} + \mathbf{G}_{hkl}$$

$\mathbf{G}_{hkl}$ is the general reciprocal lattice vector, characterised by a set of three integers *h,k,l*. Explain how the scattering formula above is equivalent to the Bragg formula for reflections from crystalline planes

$$n\lambda = 2d\mathrm{Sin}\theta$$

(5 marks)

Explain why, for the allowed (*h,k,l*) reflections from alkali halide crystals, the *h,k,l* values must be all even or all odd.

(5 marks)

Q.2 Derive Bloch's theorem for the case of the wavefunction for an electron in a solid lattice. Assume that the solid is in the form of a rectangular block and that the wavefunctions obey periodic boundary conditions. Hence show that the wavefunctions can be described by Bloch functions:

$$\psi_{\mathbf{k}}(\mathbf{r}) = \exp(i\mathbf{k}.\mathbf{r}).u_{\mathbf{k}}(\mathbf{r})$$

where $u_{\mathbf{k}}(\mathbf{r})$ has the periodicity of the lattice.

(4 marks)

Starting with the 1–D model for free-electron wavefunctions for electrons in a metal, take the periodic nature of the potential into account by the use of perturbation theory. In particular, show how the wavefunctions and the energy levels are modified in the vicinity of the Brillouin zones. In the light of this, describe the classification of materials as insulators, conductors and semiconductors.

(6 marks)

Q.3 Briefly, describe dispersion curves (ω vs. k) for the vibrational modes in a crystalline solid, and explain the approximations inherent in the Debye formula for the density of vibrational modes:

$$\rho(\omega) = \frac{9N\omega^2}{\omega_D^3}$$

(5 marks)

The Hamiltonian for a simple vibrating lattice based on harmonic forces is given by

$$H = \sum_k \frac{\hbar\omega_k}{2}\left(a_k^+ a_k + a_k a_k^+\right)$$

and the quantum state of the lattice is described by the product state

$$|n\rangle = |n_1\rangle.|n_2\rangle.|n_3\rangle.|n_4\rangle.... = |n_1, n_2, n_3, n_4, ...\rangle$$

where n_i gives the number of phonons in the mode i. The a_k and a_k^+ operators have the property

$$a_k|n_k\rangle = n_k^{1/2}|n_k - 1\rangle, \qquad a_k^+|n_k\rangle = (n_k + 1)^{1/2}|n_k + 1\rangle$$

Using the Debye density of states, derive an expression for the molar heat capacity of such a vibrating lattice, and explain in what way it differs from the classical model of Dulong and Petit.

(5 marks)

Note: $\displaystyle\int_0^\infty \frac{x^4 e^x}{\left(e^x - 1\right)^2} dx = \frac{4\pi^4}{15}$

Q.4 Many common paramagnetic insulators are found to contain either transition metal ions or rare earth ions. Explain why this is so.

(2 marks)

Describe the classical Langevin theory of paramagnetism, and show how it leads to the Curie law for the magnetic susceptibility:

$$\chi_m = C/T$$

where C is the Curie constant. Derive an expression for C from this theory.

(5 marks)

Indicate briefly how the theory would need to be modified to take account of quantum effects.

(3 marks)

Q.5 Answer *both* of the following questions.

(a) Explain why the contribution of conduction electrons to the molar heat capacity of metals at normal temperatures is so small.

Note: $$\int_0^{\infty} \frac{x^{\ell}}{e^{x-x_o}+1}\,dx \cong \frac{x_o^{\ell+1}}{\ell+1}\left[1+\frac{\pi^2\ell(\ell+1)}{6x_o^{2}}\right]$$

(5 marks)

(b) Discuss radiative processes involving electrons and holes in both direct and indirect bandgap semiconducting materials. (5 marks)