OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

GX2787

SEMESTER II

SUMMER EXAMINATIONS 2000/2001

Second University Examination in Information Technology

LOGICAL FOUNDATIONS OF COMPUTING (CT214)

Professor James Wiegold;
Professor David Bell;
Professor M.L. Newell;
Professor G. Lyons;
Dr N. Madden;
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Time allowed: **Three hours.**

Candidates should attempt four questions, two from each section.
Please use separate answer books for each section.
All questions carry equal marks.

SECTION A

A1. (i) Describe the NOT, AND and OR gates.

What does it mean that a set of gates is (functionally) **complete**. For each of the following sets, show that they are complete:

(a) $\{OR, NOT\}$

(b) $\{NAND\}$, which is defined as $(x \uparrow y) = \ \sim (x \wedge y)$.

(ii) Construct the logic table for the Boolean expression

$$(x_1 \rightarrow x_2) \wedge \overline{x}_3.$$

Write down the corresponding disjunctive normal form (DNF).

Explain briefly why the DNF takes the value 1 precisely when the given Boolean expression has value 1.

(iii) Explain what is meant by saying that a proposition is a **tautology**, or that it is a **contradiction**.

For each of the following compound propositions, determine if it is a tautology, a contradiction, or neither:

(a) $a \wedge \overline{a}$

(b) $a \vee \overline{a}$

(c) $(a \wedge b) \rightarrow a$

P.T.O.

A2. (i) What is the definition of a **valid argument**?

When is a collection of (compound) propositions **inconsistent**?

For each of the following arguments, determine if it is valid:

(a) $a \to b, b \to c \therefore a \to c$

(b) $a \to \overline{b}, b \to \overline{c} \therefore \overline{a} \to \overline{c}$

(ii) When is an instance of a valid argument **sound**?

When is an instance of a valid argument **unsound**?

Give an example of each case based on an argument from part (i).

(iii) An airline has a rule: to get a discount fare one must be at most 26 years old. When questioned, four customers answered (respectively):

(a) I am 20 years old (b) I got a discount

(c) I did not get a discount (d) I am 30 years old.

Who must be quizzed further in order to establish that the rule was applied correctly? Justify your answer.

If the airline changes to age limit to 30, and the four customers answered as above, who, if any, must now be quizzed further?

A3. (i) Describe the **tableau** method for testing a collection of well-formed formulae (compound propositions) for (in)consistency.

(ii) Use the tableau method to show that the following collection of well-formed formulae (WFFs) is consistent:

$$p \to (q \vee r), \qquad q \to p, \qquad \sim r.$$

Read off from the tableau an assignment of values to p, q and r which makes all three WFFs take the value 1.

(iii) An examiner reasons:

You will not pass this course if you do not score at least 40% on the written paper.

If you do not attended enough lectures, you will not pass the course.

You have attended enough lectures and scored at least 40% on the written paper.

Therefore you will pass the course.

Using:

"p" to represent "you will pass the course",

"q" to represent "you scored at least 40%", and

"r" to represent "you have attended enough lectures",

construct an argument underlying the reasoning.

Use *resolution* to check if the argument is valid.

P.T.O.

SECTION B

B1. (i) Using laws of the propositional calculus, prove the following:

(a) $p \vee (p \wedge q) = p \wedge (p \vee q)$

(b) $p \wedge \sim (p \wedge q) = p \wedge \sim q$

(c) $(p \wedge q) \Rightarrow r = p \Rightarrow (q \Rightarrow r)$

(d) $p \Rightarrow (q \Rightarrow r) = (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

(ii) Prove that a possible alternative definition for implication is $(p \Rightarrow q) =_{def} (q \equiv (p \vee q))$. Use the definition for equivalence:

$$(p \equiv q) =_{def} (p \wedge q) \vee (\sim p \wedge \sim q)$$

(iii) Use the propositional calculus to show that the following argument is valid: If the specification is clear and the resources are available then the programming team will be happy. The specification is clear, but the programming team is not happy. So, the resources are not available.

B2. (i) Use the following definitions:

$$(p \equiv q) =_{def} (p \wedge q) \vee (\sim p \wedge \sim q)$$

$$(p \not\equiv q) =_{def} ((p \wedge \sim q) \vee (\sim p \wedge q)$$

to prove:

(a) $(p \equiv 1) = p$

(b) $(p \not\equiv 0) = p$

(c) $\sim (p \not\equiv q) = (p \equiv q)$

(d) $(p \not\equiv q) = (\sim p \equiv q)$

(ii) Argue that the proposition

$$p_1 \not\equiv p_2 \not\equiv \ldots \not\equiv p_n$$

is true (1) if and only if an odd number of the $p_1, p_2, \ldots, p_n$ are true.

(iii) On a certain island of knights and knaves, knights always tell the truth and knaves always lie. You ask B "are you a knight?". He replies, "If I am a knight then I'll eat my hat". Prove that B has to eat his hat.

P.T.O.

B3. (i) Represent the following statements using predicate calculus:

(a) In every project team there is at least one systems analyst.

(b) Every project team has the same systems analyst.

(c) There is no employee who is a member of both team1 and team2.

Assume that E is the universe of employees, PT is the universe of project teams, and J is the universe of job titles. You may use the following atomic predicates:

- $IsIn(t, p)$ indicates whether employee p is part of project team t.
- $EmployedAs(j, p)$ indicates whether employee p has job title j.

(ii) Let U_1 be the universe of second year IT students, and U_2 be the universe of subjects in second year IT. Let $P(p, s)$ denote that student p passes subject s. What do the following mean?

(a) $\forall p : U_1 \bullet \forall s : U_2 \bullet P(p, s)$

(b) $\forall p : U_1 \bullet \exists s : U_2 \bullet P(p, s)$

(c) $\exists s : U_2 \bullet \forall p : U_1 \bullet P(p, s)$

(iii) Determine if the following argument is valid: Dilly loves all and only those who love Milly. Milly loves all and only those who do not love Dilly. Dilly loves herself. Therefore Milly loves herself.