



National University of Ireland, Galway
Ollscoil na hÉireann, Gaillimh

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THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER 1 EXAMINATIONS, 2001/02

EC501 MICROECONOMIC THEORY

M.Econ.Sc. (Economic Policy Evaluation and Planning)

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Time allowed 2^{1/2} hours.

Answer any FOUR questions.

1. Suppose that strict preference \succ is asymmetric and negative transitive, and weak preference \succeq and indifference \sim are defined as follows:

$x \succeq y$ if it is not the case that $y \succ x$,

$x \sim y$ if $x \not\succ y$ and $y \not\succ x$.

- (a) Prove that weak preference \succeq is complete and transitive.
- (b) Prove that indifference \sim is reflexive, symmetric and transitive.
- (c) Establish that if $w \sim x$, $x \succ y$, and $y \sim z$, then $w \succ y$ and $x \succ z$.

2. Given preferences \succ on a set X , a numerical representation for those preferences is any function U with domain X and range the real line such that

$x \succ y$ if and only if $U(x) > U(y)$.

- (a) Prove that for \succ to admit a numerical representation, it is necessary that \succ is asymmetric and negatively transitive.
- (b) Suppose that U is a numerical representation of preferences \succ . Establish that U is quasi-concave if and only if preferences \succ are convex.

3.
 - (a) In Bertrand-Nash competition, each firm i faces the market demand $q_i = 200 - p_i - (p_i - \text{average } p)$, $i = 1, 2$. Each firm has constant marginal cost of €10. Find the Bertrand-Nash equilibrium.
 - (b) Suppose that each firm i faces the same market demand and cost conditions as in part (a), only this time firms move sequentially. Assume that firm 1 moves first. Find the Bertrand-Stackelberg equilibrium. How does it compare to the equilibrium when both firms move simultaneously as in part (a)?
 - (c) Solve for the Cournot-Nash equilibrium of the market game given in part (a). [Hint: Write the demand system so that price is a function of quantity]. How does it compare to the equilibrium of part (a)?
4.
 - (a) Suppose that there are n firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i and let $q = q_1 + q_2 + \dots + q_n$ denote the aggregate quantity on the market. Let p denote the market-clearing price and assume that inverse demand is given by $p = 260 - q/2$.

Suppose that the constant marginal cost for all firms is €20 and there are no fixed costs. What is the Cournot-Nash equilibrium?
 - (b) Suppose now that there is a fixed cost (F) in addition to the €20 marginal cost. If $F = €4,800$, and all firms regard the Cournot model as the correct oligopoly model, how many firms will enter the market (assuming that entry is sequential)? What profit will each firm earn?
 - (c) Again suppose that $F = €4,800$, and that all firms regard the collusive model as the correct oligopoly model, and that entry is sequential. How many firms will enter the market? What profit will each firm earn?
 - (d) Finally, assume that $F = €4,800$ and that all firms regard the Bertrand model as the correct oligopoly model, and that entry is sequential. How many firms will enter the market? What profit will each firm earn?
 - (e) Briefly comment on the significance of different oligopoly model assumptions for entry deterrence using (b), (c) and (d) above.

5. Suppose that the inverse market demand function is given by

$$p = 130 - q/2.$$

The constant marginal cost for all firms is €10.

- (a) Find the monopoly output and price, and compute the monopolist's profit. Then find the Cournot-Nash equilibrium output for each firm and the aggregate output and price in the Cournot-Nash equilibrium, and compute the equilibrium profit of a representative firm when there are two firms in the market.
 - (b) Suppose that there are two firms in the market. What is the symmetric collusive equilibrium? How much profit does each firm earn in this equilibrium? If one firm honours the collusive agreement, but the other violates the agreement, how much should the violator produce to maximise its profit? What is the magnitude of the inducement to cheat on the collusive agreement? Does it exceed the losses of the other firm?
 - (c) Find a subgame perfect Nash equilibrium that supports monopoly payoffs for the Cournot market game of part (a) infinitely repeated. Discuss the role of interest rates/discount factor.
6. Given a preference relation \succ on a set X and non-empty subset A of X , the set of acceptable alternatives from A according to \succ is defined to be $c(A, \succ) = \{x \in A : \text{there is no } y \in A \text{ such that } y \succ x\}$.
- (a) Prove that for any \succ , $c(A, \succ) \subseteq A$.
 - (b) Assume that \succ is asymmetric and negatively transitive. Establish that for every finite set A , $c(A, \succ)$ is not empty.
 - (c) Suppose that both x and y are in both A and B , and $x \in c(A, \succ)$ and $y \in c(B, \succ)$. Establish then that $x \in c(B, \succ)$ and $y \in c(A, \succ)$.