National University of Ireland, Galway
Ollscoil na hÉireann, Gaillimh

Winter Examinations Semester I 2001/2002

EC223 Introduction to Mathematical Economics

Prof. Michael Cuddy
Prof. Huw Dixon
Dr Aidan Kane

Please answer SIX of the following questions. All questions carry equal marks.

Time allowed: **TWO hours**

1. For each of the following functions $y = f(x)$, find the first derivative $\frac{dy}{dx}$

   (a) $y = x^3 + \frac{1}{x^4}$

   (b) $y = x^3 - x^{-2} + 7x$

   (c) $y = 3x^{-4} - 2\sqrt{x}$

   (d) $y = 3x^2(10 + x^3)$

   (e) $y = (x^2 + 3x)(3x + \frac{3}{x^2})$

   (f) $y = \frac{(x^3+7x-3x)}{(x^2+1)}$

   (g) $y = (x^2 + 3x^2 + 5)^9$

2. Given a demand function of the form

$$P = aQ + b$$

   where $a < 0$ and $b > 0$ (both are constants), find expressions for, and draw graphs of:

   (a) Total revenue

   (b) Marginal revenue

   (c) Average revenue

3. Given that total revenue, TR, is simply PQ, use the product rule to show that marginal revenue is given by:

$$MR = P\left(1 - \frac{1}{\epsilon}\right)$$

   where ϵ is the price elasticity of demand.

4. Find all the (first-order) partial derivatives for the following functions:

   (a) $y = 4x_1^2 + 6x_1x_2^3 + x_1^5x_2x_3^3$
   (b) $Y = 0.5K^{0.3}L^{0.7}$
   (c) $q = Ak^{\alpha}l^{1-\alpha}$ where $0 < \alpha < 1$, is a constant, and A is also a constant.

5. A firm's production function is $q = Ak^{\alpha}l^{\beta}$ where q is output, k is capital, l is labour, $\alpha, \beta > 0$. Assume that the firm gets a price for a unit of its output, p, that the price of a unit of capital is r, and that the price of a unit of labour is w. Find the profit maximising levels of k, l and q for this firm.

6. A consumer maxmizes her utility function $U = (x_1 + 1)(x_2 + 2)$ subject to the usual constraint $M = p_1x_1 + p_2x_2$. Solve this problem for x_1^*, x_2^* and the Lagrange multiplier λ^*. Interpret λ^* for specific values of p_1, p_2 and M.

7. Using matrix methods, solve the following equations for x_1 and x_2:

$$\begin{aligned} 8x_1 - 7x_2 &= -6 \\ 3 &= -x_1 - x_2 \end{aligned}$$

8. Given the matrices A and B below, show that B is the inverse matrix of A.

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 7 & 3 & 3 \\ 2 & 0 & 1 \end{bmatrix} \quad B = \frac{1}{8}\begin{bmatrix} 3 & 2 & -9 \\ -1 & 2 & -5 \\ -6 & -4 & 26 \end{bmatrix}$$

9. Given a demand function

$$P = 40 - 3Q$$

   find the consumer's surplus when $Q = 4$. Illustrate your result graphically.

10. Given that the flow of invetmenent $I(t) \equiv \frac{dK}{dt}$, and is given by

$$I(t) = 800t^{1/3}$$

   find the number of periods (t) required before the capital stock exceeeds 48,600.

11. An investment proposal requires and initial cost of £15,000 and will produce a return of £20,000 in three years' time. By calculating the present value of this proposal, and its internal rate of return, determine whether it represents a worthwhile investment, assuming that the interest rate is 5% compounded annually.