OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2001-2002

EC218 MATHEMATICAL ECONOMICS

Second Year

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Time allowed: TWO hours.
Answer any THREE questions.

1. A machine shop owner is attempting to decide whether to purchase a new drill press, a lathe, or a grinder. The return from each will be determined by whether the company succeeds in getting a government military contract. The profit or loss from each purchase and the probabilities associated with each contract outcome are shown in the following payoff table:

Purchase	Contract (.4)	No Contract (.6)
Drill Press	80,000	-16,000
Lathe	40,000	8,000
Grinder	24,000	20,000

(a) Compute the expected value for each purchase and select the best one.

(b) The machine shop owner now is considering hiring a military consultant to ascertain whether the shop will get the government contract. The consultant is a former military officer who uses various personal contracts to find out such information. By talking to other shop owners who have hired the consultant, the owner has estimated a .70 probability that the consultant would present a favourable report given that the contract is awarded to the shop, and a .80 probability that the consultant would present an unfavourable report given that the contract is not awarded. Using decision tree analysis, determine the decision strategy the owner should follow, the expected value of this strategy, and the maximum fee the owner should pay the consultant.

(c) Compute the efficiency of the sample information for the machine shop owner.

2. In a duopoly with homogenous product, the rival firms face the inverse market demand P = 260 - Q/2, and both have identical marginal costs of €20, where

   P = price of output
   $Q = (q_1 + q_2)$
   q_1, q_2 = output of firms 1 and 2 respectively.

   (a) Solve for the Cournot-Nash equilibrium in this model, stating clearly your assumptions, and calculate the equilibrium price, quantities and profit levels for the two firms.

   (b) Solve for the Stackelberg equilibrium in this model, assuming that firms move sequentially and that firm 1 moves first.

   (c) Explain the difference between the two types of game specified in (a) and (b) above.

3. A real estate development firm is considering several alternative development projects. These include building and leasing an office park, purchasing a parcel of land and building an office-building to rent, buying and leasing a warehouse, building a shopping centre, and building and selling condominiums. The financial success of these projects depends on interest rate movement in the next five years. The various development projects and their five-year financial return ($ millions) given that interest rates will decline, remain stable, or increase are shown in the following payoff table:

	Interest Rates		
Project	**Decline**	**Stable**	**Increase**
Office park	1.0	3.4	9.0
Office building	3.0	3.8	4.8
Warehouse	3.4	2.8	2.0
Shopping centre	1.4	4.8	7.2
Condominiums	6.4	3.0	1.2

   (a) Determine the best investment using the following decision criteria:

   (i) Maximax
   (ii) Maximin
   (iii) Equal likelihood
   (iv) Hurwicz ($\propto = .3$)

(b) The real estate development firm has decided to hire an economist to assign a probability to each direction interest rates may take over the next five years. The economist has determined that there is a .30 probability that interest rates will decline, a .40 probability that rates will remain stable, and a .60 probability that rates will increase. Using expected value, determine the best project.

(c) Determine the expected value of perfect information.

4. The following transition matrix describes the accounts receivable process for the ABC Department Store:

	p	1	2	b
p	1	0	0	0
1	.70	0	.30	0
2	.40	0	0	.60
b	0	0	0	1

The states p and b represent an account that is paid and a bad account respectively. The numbers 1 and 2 represent the fact that an account is either one or two months overdue. After an account has been overdue for two months, it becomes a bad account and is transferred to the store's overdue accounts section for collection. The company has sales of €630,000 each month. Determine how much the company will be paid and how many of the debts will become bad debts in a two-month period.

5. In a quantity setting duopoly with homogenous product, the inverse market demand function is given by:

$P = 400 - Q$, where
$Q = (q_1 + q_2)$
q_1, q_2 = output of firms 1 and 2 respectively.

The constant marginal cost for each firm is 40.

(a) Find the collusive outcome where the firms act as a joint monopolist. How does it differ from the Cournot-Nash equilibrium?

(b) Can a subgame perfect Nash equilibrium that supports monopoly payoffs be achieved for this Cournot market game infinitely repeated. Analyse the role of the interest rate or discount factor in achieving the equilibrium.

(c) Comment on the likelihood and sustainability of collusion when the number of firms in the industry grows significantly.