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THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

AUTUMN EXAMINATIONS 2001-2002

EC381 Game Theory and Industrial Organisation
Third Year

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Time allowed: TWO hours.

Answer THREE questions.

1.
 - (a) In Bertrand-Nash competition, each firm i faces the market demand $q_i = 200 - p_i - (p_i - \text{average } p)$, $i = 1, 2$. Each firm has constant marginal cost of €10. Find the Bertrand-Nash equilibrium.
 - (b) Assume that each firm i faces the same market demand and cost conditions as in part (a). Only this time firms move sequentially. Let us say that firm i moves first. Find the Bertrand-Stackelberg equilibrium. How does it compare to the equilibrium when both firms move simultaneously as in part (a)?
 - (c) Solve for the Cournot-Stackelberg equilibrium of the market game given in part (b). (Hint: Write the demand system so that price is a function of quantity). How does it compare to the equilibrium of part (b).
2.
 - (a) Suppose that there are n firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i , and let $Q = q_1 + q_2 + \dots + q_n$ denote the aggregate quantity on the market. Let P denote the market clearing price and assume that inverse demand is given by $P = 260 - Q/2$.

Suppose that the constant marginal cost for all firms is €20 and that there are no fixed costs. What is the Cournot-Nash equilibrium?
 - (b) Suppose now that there is a fixed cost (F) in addition to the €20 marginal cost. If $F = €4,800$, and all firms regard the Cournot model as the current oligopoly model, how many firms will enter the market (assuming that entry is sequential)? What profit will each firm earn?
 - (c) Again assume that $F = €4,800$ and that all firms regard the collusive model as the correct oligopoly model and that entry is sequential. How many firms will enter the market? What profit will each firm earn?

- (d) Finally, suppose that $F = €4,800$ and that all firms regard the Bertrand model as the correct oligopoly model, and that entry is sequential. How many firms will enter the market? What profit will each firm earn?
- (e) Briefly comment on the significance of different oligopoly model assumptions for entry deterrence using (b), (c) and (d) above.
3. Consider the Dixit capacity investment model. When the inverse market demand function is given by: $p = 360 - q$, $w = 20$, $r = 40$ and $K = 5,625$, where p is a price per unit, w and r are input prices and k is entry cost.
- (a) Derive a new firm's post-entry best response function
- (b) Given initial capacity of x_1 , derive the incumbent firm's post-entry best response function
- (c) Given x_1 , derive the post-entry equilibrium profit of a new firm
- (d) Derive the minimum capacity that makes entry unprofitable and determine the optimal capacity choice of the incumbent firm.
4. The inverse market demand function for a duopoly is given by:
- $$p = 180 - q/2$$
- The constant marginal cost for each firm is €60.
- (a) State and explain the Cournot-Nash equilibrium for this model. How does this equilibrium differ from the case where the firms act as a joint monopolist? What is the magnitude of the inducement to cheat on the collusive agreement?
- (b) Now suppose that there is a fixed cost (F) in addition to the €60 marginal cost. If $F = €1,250$, what is the limit output q_L . Supposing that q_L effectively deters entry, what profit would a monopolist earn by using q_L ?
- (c) Critically evaluate the role of the Sylos postulate in your answer to part (b).
5. In the static Bertrand duopoly model (with homogeneous product), the firms name prices simultaneously. The demand for the firm i 's product is $100 - p_i$, if $p_i < p_j$, is 0 if $p_i > p_j$, and is $(100 - p_i)/2$ if $p_i = p_j$. Marginal costs are constant at €40. Consider the infinitely repeated game based on this stage game. Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium. Discuss the role of interest rate/discount factor.