6x460

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

Autumn Examinations 2002

Techniques of Analysis (EC222)

2nd B.A. (Economic and Social Studies) – St. Angela's College, Sligo

Prof. Michael Cuddy
Prof. Huw Dixon
Ms. Breda Lally

Time Allowed: **Two Hours**
Marks: **300**

Instructions: This exam consists of two sections. You must answer questions from both sections. Please read carefully the instructions for each section.

- **Section A (150 Marks):** **Answer FOUR questions. You MUST answer Question 1 OR Question 2 (worth 45 marks each) AND THREE other questions (worth 35 marks each)**
- **Section B (150 Marks):** **Answer THREE questions. You MUST answer Question 1 (worth 30 marks) AND TWO other questions (worth 60 marks each)**

Section A

1. The owner of a restaurant serving continental food wants to study characteristics of customers of her restaurant. In particular, she decides to focus on two variables: the amount of money spent by customers and whether or not customers order dessert. The results from a sample of 60 customers are as follows:

   - Amount spent: $\bar{X} = €38.54$, $S = €7.26$
   - 18 customers purchased dessert

   a. Set up a 95% confidence interval estimate of the population average amount spent per customer in the restaurant.
   b. Set up a 90% confidence interval estimate of the population proportion of customers who purchase dessert.
   c. Assuming the owner of a competing restaurant wishes to conduct a similar survey in her restaurant (and does not have access to the information obtained by the owner of the first restaurant) what sample size is needed if she wishes to be 95% confident of estimating the true population average amount spent in her restaurant to within ± €1.50 and the standard deviation is assumed to be €8.

2. Plastic bags used for packaging produce are manufactured so that the breaking strength of the bag is normally distributed with a mean (μ) of 5 pounds per square inch and a standard deviation (σ) of 1.5 pounds per square inch.
   a. What proportion of the bags produced have a breaking strength between 3.2 and 4.2 pounds per square inch?
   b. If a sample of 25 bags is selected what is the probability that the average breaking strength is less than 4.6 pounds per square inch?
   c. Between what two values symmetrically distributed around the mean will 95% of the average breaking strengths fall?

3. The finance society at a college of business at a large state university would like to determine whether there is a relationship between a student's interest in finance and his or her ability in mathematics. A random sample of 400 students is selected and they are asked whether their interest in finance and ability in mathematics are low, average or high. The results are as follows:

	Ability in Mathematics			
Interest in Finance	**Low**	**Average**	**High**	**Total**
Low	10	90	50	150
Average	30	20	30	80
High	120	30	20	170
Total	160	140	100	400

If a student is selected at random, what is the probability that he or she
   a. has a low ability in mathematics?
   b. has a high interest in finance?
   c. has a high interest in finance and a high ability in mathematics?
   d. has an average ability in mathematics and an average interest in finance?
   e. has an average ability in mathematics or a high interest in finance?
   f. given that the student selected has a high interest in finance what is the probability that he or she has a low ability in mathematics?

4. The probability of a football player scoring a goal is 0.3. He takes 6 shots at goal. Find the probability that he scores
   a. three goals
   b. at least three goals
   c. less than three goals

5. A potential entrepreneur is considering the purchase of a coin-operated laundry. The present owner claims that over the past five years the average daily revenue has been €675 (μ) with a standard deviation of €75 (σ). A sample of 30 selected days reveals a daily average revenue of €625. Using the 0.05 level of significance, is there evidence that the claim of the present owner is not valid?

6. On the basis of industry sales of $1.5 billion recorded for the 1-year period ending May 25, 1997, The New York Times reported (June 20, 1997) that Crest toothpaste was the market leader with a share of 26.3%. Suppose that a recently taken random sample of 250 individuals indicates that 68 are using Crest toothpaste. At the 0.05 level of significance, is there evidence that the proportion has changed from the previous 1996-1997 market share?

7. Suppose that the manager of a paint supply store wants to estimate that actual amount of paint contained in 1-gallon cans purchased from a nationally know manufacturer. It is known from the manufacturer's specifications that the standard deviation (σ) of the amount of paint is equal to 0.02 gallon. A random sample of 50 cans is selected, and the average amount of paint per 1-gallon can is 0.995 gallon. Set up a 99% confidence interval estimate of the true population average amount of paint included in a 1-gallon can.

8. A set of final examination grades in an introductory statistics course was found to be normally distributed with a mean of 73 and a standard deviation of 8.
   a. What percentage of students scored between 65 and 89?
   b. Only 5% of the students taking the test scored higher than what grade?

Section B

1. Consider the following basic Keynesian model of the macro economy

$$Y = C + I_0 + G_0$$
$$C = a + bY$$

Where Y and C are endogenous, I_0 and G_0 are exogenous constants, $a>0$, and $0<b<1$. Use either matrix inversion or Cramer's Rule to solve for Y and C.

2. The demand and total cost functions for a good are given by the equations:

$$P = 100 - 4Q$$
$$TC = 320 + 4Q$$

   a. Write down the equations for total revenue (TR), marginal revenue (MR), average cost (AC) and profit (π).
   b. Determine the number of goods that must be produced and sold to maximise profit. Hence calculate maximum profit.

3. Using matrix inversion or Cramer's Rule, solve the following equations for x, y and z.

$$x = 12 - y$$
$$5y + 2z = 20 - 2x$$
$$6x + 3y + 6z = 0$$

4. A firm has a total cost function:

$$TC = 30Q - 15Q^2 + 3Q^3$$

   a. Write down the equations for average cost (AC) and marginal cost (MC).
   b. Determine the values of Q at which (i) MC and (ii) AC are minimised.
   c. Confirm algebraically that AC and MC are equal when AC is at a minimum.

5. Find all the first and second order partial derivatives for the following functions:

   a. $y = 10a^2 + 6ab + 4b^2$
   b. $Q = 10L^{0.5}K^{0.5}$