Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX 692

Semester II Examinations, 2002/2003

Exam Code(s)	4BN121, 4BP121
Exam(s)	B.E. Degree in Electronic Engineering B.E. Degree in Electronic and Computer Engineering
Module Code(s)	EE409
Module(s)	Digital Signal Processing
Paper No.	1
Repeat Paper	Special Paper
External Examiner(s)	Prof. S. McLaughlin
Internal Examiner(s)	Prof. D.J. Wilcox Dr. E. Jones

Instructions:

Answer 5 questions.
All questions carry equal marks.

Duration	3 hrs.
No. of Answer books	

Requirements:

Handout	
MCQ	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	

No. of Pages	6
Department(s)	Electronic Engineering

1. (a) Determine the impulse response of the discrete-time system shown in Figure 1(a).

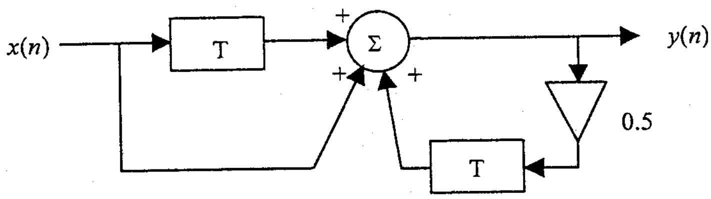

Figure 1(a)

[6 marks]

(b) Let $h_1(n)$ and $h_2(n)$ be defined as follows:

$$h_1(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & otherwise \end{cases} \qquad h_2(n) = \begin{cases} 2-n & 0 \le n \le 2 \\ 0 & otherwise \end{cases}$$

Compute the impulse response of a system consisting of the cascade of $h_1(n)$ and $h_2(n)$.

[6 marks]

(c) Two filter structures are shown in Figures 1(b) and 1(c):

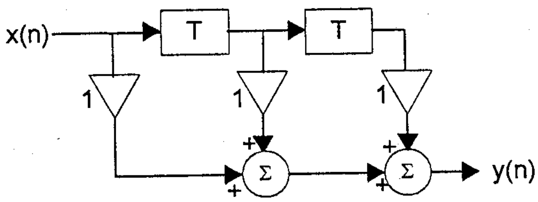

Figure 1(b)

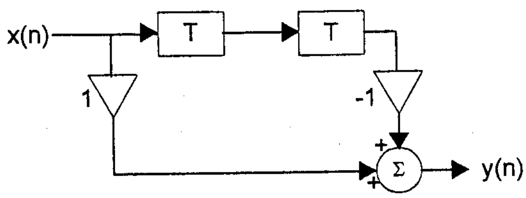

Figure 1(c)

Show that both filters have linear phase characteristics, and explain why linear phase is important in some signal processing applications.

[8 marks]

2. (a) Give the equations for the Discrete Fourier Transform (DFT), and the inverse DFT, and explain how the magnitude spectrum of a signal may be obtained in this way.

Explain what is meant by windowing, and how and why it is used in spectral analysis of discrete-time signals. Give the equation, and describe the characteristics and typical application for each of the following windows:
(i) the Hamming window of length N;
(ii) the Kaiser window of length N.

[6 marks]

(b) A discrete-time system has impulse response

$$h(n) = \alpha^n u(n)$$

Obtain an expression for the frequency response of the system, and hence calculate the phase spectrum. From the phase spectrum, obtain an expression for the group delay of the system.

[cont'd]

Use the fact that

$$\frac{d}{dx}\tan^{-1}\left[\frac{-a\sin x}{1-a\cos x}\right] = -\frac{a^2 - a\cos x}{1+a^2-2a\cos x}$$

[6 marks]

(c) The Fourier transform of the sequence $a^n u(n)$ is given by:

$$\frac{1}{1-ae^{-j\theta}}$$

Using this fact, find the output of a system with impulse response $h(n) = a^n u(n)$, when the sequence $b^n u(n)$ is input to the system (hint: recall the convolution property of the Fourier transform).

[8 marks]

3. (a) Design a digital oscillator to produce a sustained oscillation according to the equation:

$$p(n) = \cos(n\theta_0)$$

where θ_0 is the frequency of oscillation. Indicate how the coefficients of the oscillator (b_1 and b_2) are calculated (and thus how the oscillation frequency may be controlled), and determine the initial conditions required to start the oscillation without any input signal. Draw a block diagram of the oscillator.
[Hint: start from the z-transform of $p(n)$].

[9 marks]

(b) If b_1=2cosθ_0, show that the frequency resolution of the oscillator is given by:

$$\Delta f_o = \frac{-f_s \Delta b_1}{4\pi \sin\left(\frac{2\pi f_o}{f_s}\right)}$$

where f_s is the sampling frequency (16 kHz) f_o is the desired frequency of oscillation, and Δf_0 is the change in oscillation frequency caused by a change Δb_1 in the coefficient b_1. For a fixed-point implementation, calculate the highest frequency resolution that is obtainable for this oscillator. You may assume that b_1 is represented by a K-bit number where K=16 (using fractional arithmetic), and Δb_1 is given by:

$$\Delta b_1 = \frac{1}{2^{K-2}}$$

[11 marks]

4. (a) Using the pole-zero placement method, obtain the transfer function of a digital band-pass filter with the following characteristics:
(i) Sampling rate of 3000 Hz
(ii) Complete rejection of the input signal at DC and half the sampling frequency
(ii) A narrow passband centred at 300 Hz, with a 3 dB bandwidth of 25 Hz.

Sketch the pole-zero map, and give the difference equation of the filter.

[6 marks]

[cont'd]

(b) Using the Impulse Invariant Transformation, design a digital filter based on the following analog transfer function:

$$H(s) = \frac{3}{(s+2)(s+3)}$$

[7 marks]

(c) Using the bilinear transformation, determine the transfer function of the digital equivalent of a first-order RC analogue filter, whose normalised (i.e. $\theta_c = 1$ rad/s) transfer function is given by:

$$H(s) = \frac{1}{s+1}$$

Assume a sampling frequency of 4 kHz, and a cutoff frequency of 500 Hz.

[7 marks]

5. (a) A signal processing application requires that the sampling frequency for a particular signal must be changed from 2 kHz to 3 kHz. Design a rate-changing system to achieve this, and sketch approximately the spectra of the signals at various points in the system. Also, indicate the cutoff frequency of any filters required (assume that "ideal" low-pass filters are used). You may assume that the bandwidth of the original signal is 800 Hz, and you may choose any arbitrary spectrum.

Comment on the nature of the filters you would use (i.e. FIR or IIR, phase characteristics).

[8 marks]

(b) Figure 5 shows a block diagram of a signal processing system consisting of the following elements:
(i) An Analogue-to-Digital converter (A/D), with a sampling period of 0.4 msec.
(ii) A demodulator, wherein the A/D output signal $x(nT_1)$ is multiplied by a cosine carrier with a frequency of oscillation of 1 kHz.
(iii) A low-pass filter, with a cutoff frequency of 125 Hz
(iv) A downsampler, whose function is to reduce the sampling rate of its input signal by a factor of 10.

The input signal to the A/D is a bandpass signal, whose passband extends from 875 Hz to 1125 Hz.

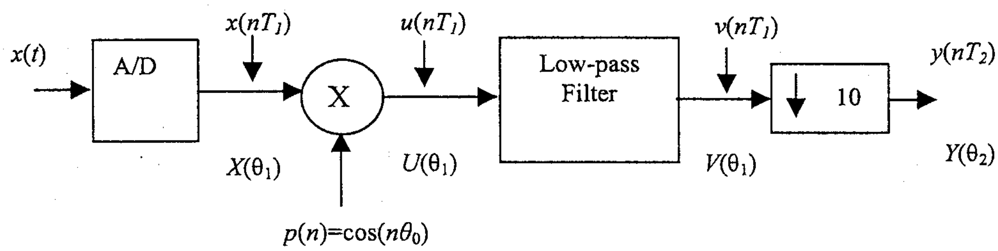

Figure 5.

You are required to:
(i) Determine the sampling frequency each of the signals, $x(nT_1)$, $u(nT_1)$, $v(nT_1)$ and $y(nT_2)$.
(ii) Sketch, one above the other, the magnitude of each of the spectra $X(\theta_1)$, $U(\theta_1)$ and $V(\theta_1)$, against θ_1.
(iii) Sketch the magnitude of $Y(\theta_2)$ against θ_2.

Note: You should watch for the possibility of aliasing occurring in the demodulator.

[12 marks]

6. (a) In the speech production model, a commonly used approximation for the impulse response of the glottal pulse-shaping filter is as follows:

$$g(n) = na^n, n \geq 0$$
$$g(n) = 0, n < 0$$

By obtaining the transfer function of this filter, choose a value of a such that the the magnitude response at DC is 60 dB higher than the response at half the sampling frequency.

[7 marks]

(b) A speech utterance has a duration of 1 second, and consists of the following segments:

- an initial period of silence 100 msec long;
- a period of voiced speech 200 msec long;
- a period of unvoiced speech 400 msec long;
- a second period of voiced speech 100 msec long;
- a final silent period 200 msec long.

Sketch the general form of:
(i) The Energy contour
(ii) The Zero-Crossing Count contour

[6 marks]

(c) Briefly describe the basis of the Linear Predictive Coding method of speech analysis (there is no need to do a detailed derivation of the equations). Discuss the relationship between the order of the prediction filter and the prediction error.

[7 marks]

7. (a) Give the equation for the Least Mean Squares (LMS) algorithm for adaptive filtering, and discuss the stability and convergence of filters employing this algorithm. In particular, indicate how the step-size should be chosen, and comment on the influence of algorithm step-size on the stability, convergence speed, and steady-state performance of the algorithm.

[6 marks]

(b) An adaptive echo canceller is required as part of a transceiver in a high-speed datacommunications system. The input data to the echo canceller is a random sequence of data symbols, with values taken from the following set [-7, -5, -3, -1, 1, 3, 5, 7]. All values are assumed equiprobable. The sampling rate of the system is 200 kHz, and the length of the echo path is approximately 1.6 milliseconds.

(i) Calculate the minimum number of taps needed in the echo canceller.
(ii) Assuming the Least Mean Squares (LMS) algorithm is to be used for adaptation, choose a suitable value for the adaptation step-size.
(iii) Discuss how the implementation of the echo canceller may be made more efficient, given the characteristics of the input data.

[8 marks]

(c) Describe the Normalised LMS algorithm for adaptive filtering, including the equation for the coefficient update and the applications where it might be used, and outline the advantages and disadvantages of the algorithm.

[6 marks]

Table of useful z transforms

		Sequence	z-Transform
1.	Unit sample	d(n)	1
		d(n-k)	z^{-k}
2.	Unit step	u(n)	z/(z-1)
3.	Exponential	a^n u(n)	z/(z-a)
4.	Sinusoidal	$\sin(\theta_o n)\,u(n)$	$\dfrac{z\sin\theta_o}{z^2-2z\cos\theta_o+1}$
		$\cos(\theta_o n)\,u(n)$	$\dfrac{z^2-z\cos\theta_o}{z^2-2z\cos\theta_o+1}$
5.	Unit ramp	nu(n)	$\dfrac{z}{(z-1)^2}$
6.	Product of ramp and signal	nx(n)	$-z\dfrac{dX(z)}{dz}$
7.	Sum of Series: $1+z^{-1}+z^{-2}+z^{-3}+\ldots\ldots\ldots.+z^{-(N-1)}=\dfrac{1-z^{-N}}{1-z^{-1}}$		