GX806

OLLSCOIL NA hÉIREANN

The National University of Ireland

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Fourth Year Mechanical and Biomedical Engineering Examination

FINITE ELEMENT METHODS IN ENGINEERING ANALYSIS

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Attempt Five Questions
Time allowed: 3 Hours

1. A bar composed of two sections is described in Figure 1 and is to be modelled using one truss element per section as indicated.

   (a) Compute the value of the ambient temperature rise in order that the gap at the right hand end is reduced to zero. ***(10)***

   (b) Compute the total displacement of the Node 2 at the location where the cross-section changes due to a total rise in temperature of 200^0C ***(10)***

NOTE:

$$\underline{K} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \; ; \; EA\int_0^L \underline{B}^T \underset{\sim}{\varepsilon}_0 \, dx = EA\alpha\Delta T\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

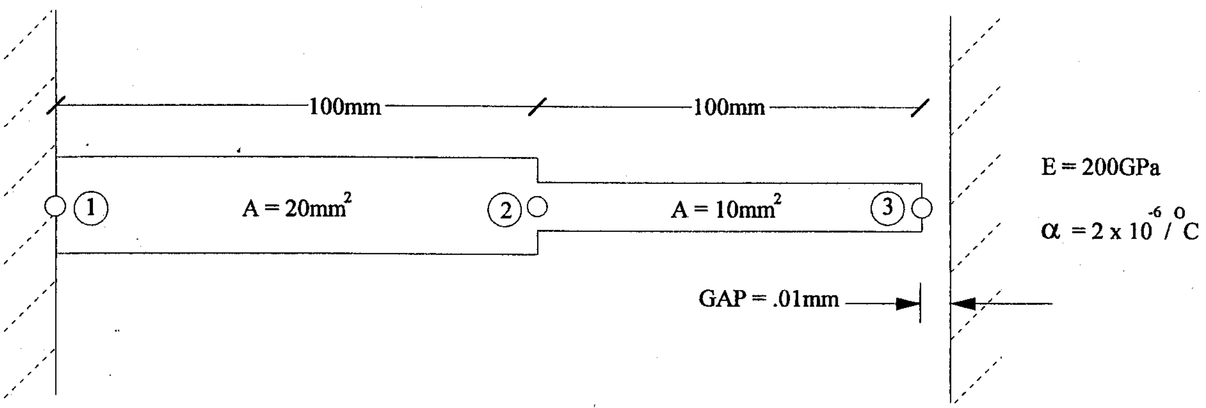

Figure 1

2. A composite wall is composed of three sections where a layer of insulation is sandwiched between two brick walls as shown in Figure 2.

You are required to develop a one-dimensional finite element solution for the temperature profile across the wall using one truss element per wall section as indicated on Figure 2 and to compute the following specific items:

(i) The temperature at Nodes 2 and 3 between the insulation and brick layers. *(10)*

(ii) The values of the heat flows at Nodes 1 and 4 on the external sides of the wall. *(10)*

Note: The form of the heat conductance finite element equations is given as

$$\frac{KA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} T_i \\ T_j \end{bmatrix} = \begin{bmatrix} Q_i \\ Q_j \end{bmatrix}$$

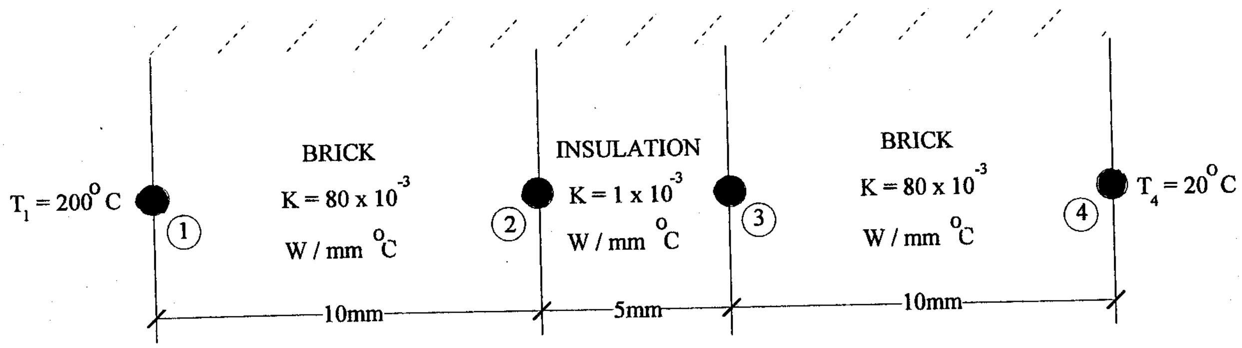

Figure 2

3. Compute the nodal loads on the axisymmetric triangular element shown in Figure 3 for the case of centrifugal loading given by $\rho\omega^2 r$ where ω is the rotational speed about the z axis and ρ is the mass density. You may take r = $\bar{r}$, the centroidal value, when developing the expression for the nodal loads. *(20)*

Note: For triangular coordinates

$$\int_A L_1^{\alpha} L_2^{\beta} L_3^{\gamma} dA = \frac{\alpha!\beta!\gamma!}{(\alpha+\beta+\gamma+2)!}2A$$

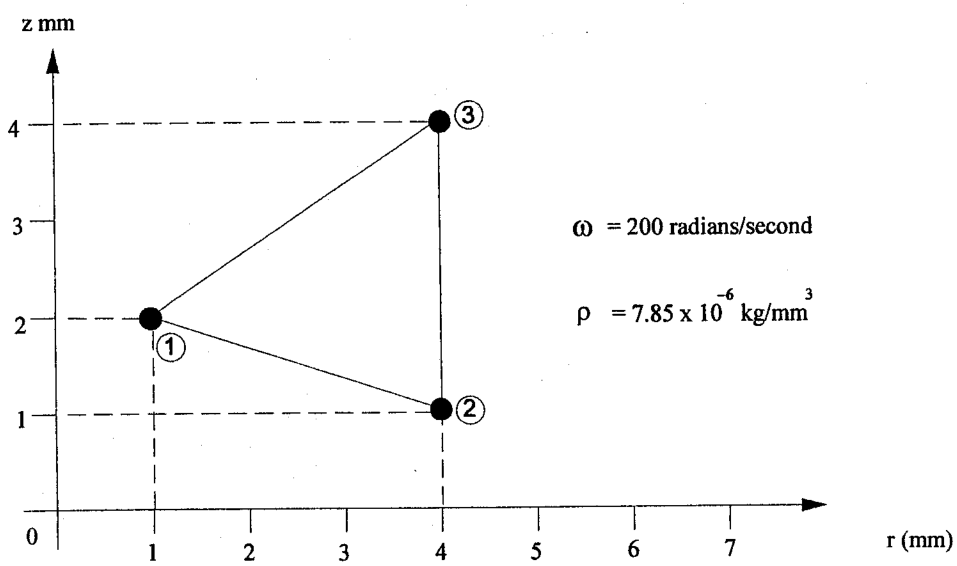

Figure 3

4. The steady state temperature ϕ of a region R with a boundary S is governed by the functional

$$T(\phi)=\frac{1}{2}\int_R\left\{K\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)-2Q\phi\right\}dr$$

$$+\int_{S_q}\left(q\phi+\frac{1}{2}h\phi^2-h\phi\phi\infty\right)ds$$

where K is the thermal conductivity,
Q is the specific internal heat generation,
h is the convection heat transfer coefficient,
q is the normal heat flux on the section S_q of S
and ϕ_∞ is the reference ambient temperature.

You are required to develop the finite element forms of the three terms under the S_q boundary integral assuming that linear triangular element shape functions are used to model the temperature distribution in R. You may assume that the section S_q of the boundary covers just one side of the triangular element. *(20)*

Note: All required integrals may be computed using the formula

$$\int_S L_1^{\alpha}L_2^{\beta}ds=\frac{\alpha!\beta!}{(\alpha+\beta+1)!}S$$

5(a) Locate the point $P(x,y)$ in the mapped domain corresponding to $P(L_1, L_2) = (\frac{1}{3}, \frac{1}{3})$ as shown in the parent domain for the simple linear triangular element shown in Figure 5. *(8)*

(b) Derive the Jacobian for the mapping in (a) above and compute the partial derivatives $\frac{\partial L_i}{\partial x}$ *and* $\frac{\partial L_i}{\partial y}$ at $P(x,y)$. Briefly comment on your result. *(12)*

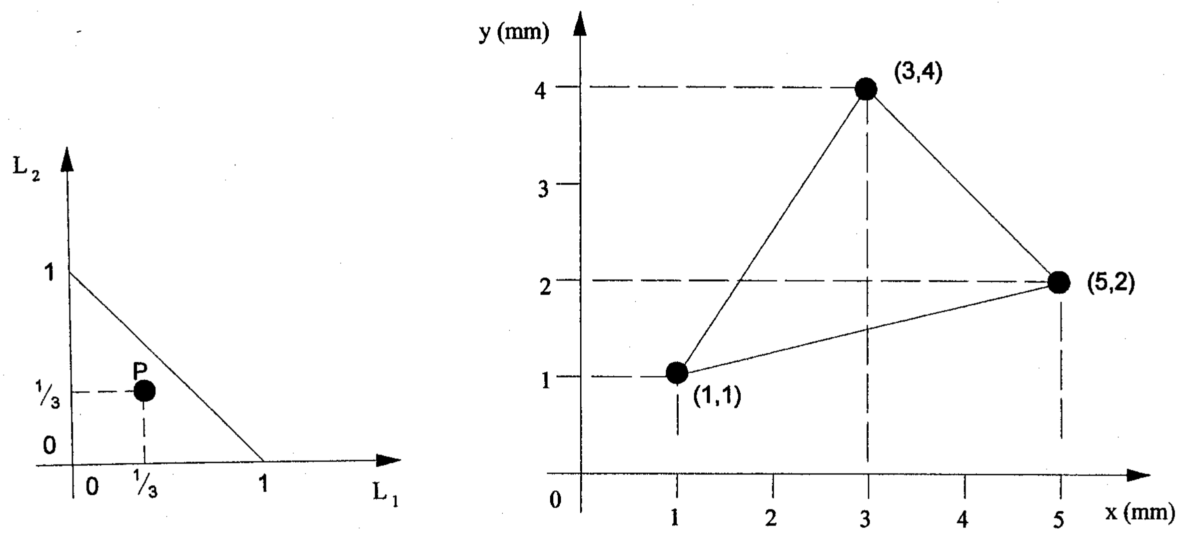

Figure 5

6. The set of nodal shape functions for the 8 – noded quadrilateral element shown in Figure 6(a) are given as follows:

Corner Nodes 1, 3, 5 and 7

$$N_i(\xi,\eta) = \frac{1}{4}\left(1+\bar{\xi}\right)\left(1+\bar{\eta}\right)\left(\bar{\xi}+\bar{\eta}-1\right)$$

Midside Nodes 2 and 6

$$N_i(\xi,\eta) = \frac{1}{2}\left(1-\xi^2\right)\left(1+\bar{\eta}\right)$$

Midside Nodes 4 and 8

$$N_i(\xi,\eta) = \frac{1}{2}\left(1-\eta^2\right)\left(1+\bar{\xi}\right)$$

where $\bar{\xi} = \xi\ \xi_L$ and $\bar{\eta} = \eta\ \eta_i$

and ξ_i and η_i are nodal coordinates

(a) You are required to show how the above expressions are derived by developing the interpolation functions for one corner and one midside node. *(8)*

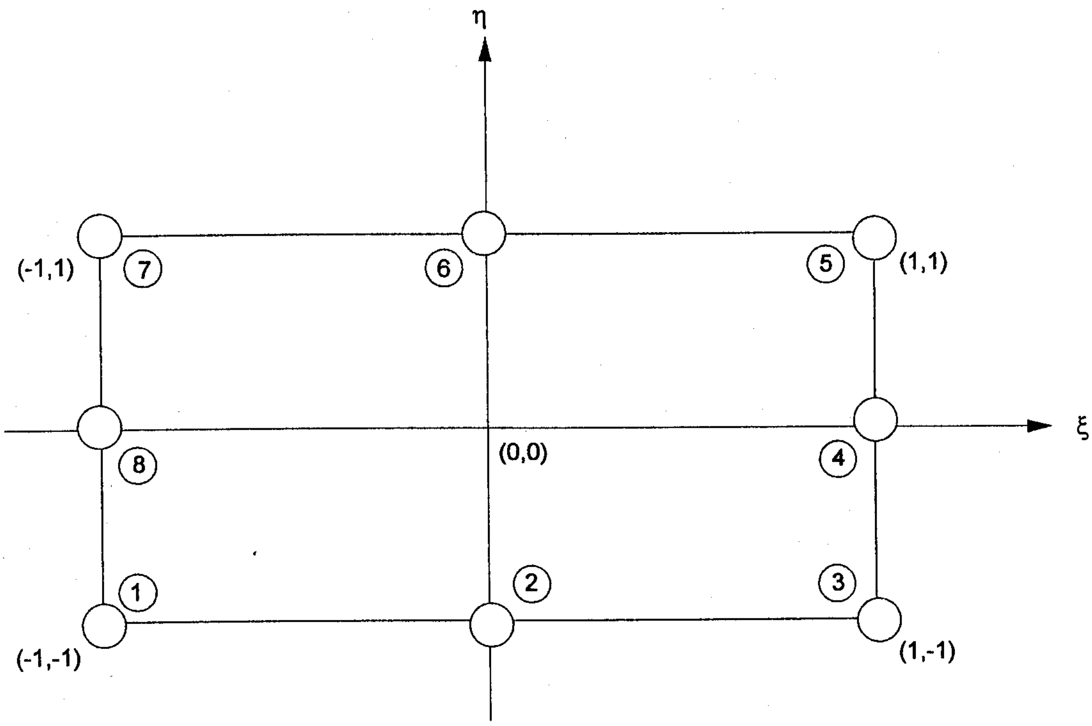

Figure 6(a)

(b) Calculate the nodal load vector for the element shown in Figure 6(b) due to the normal pressure on the side with Nodes (3), (4) and (5). *(12)*

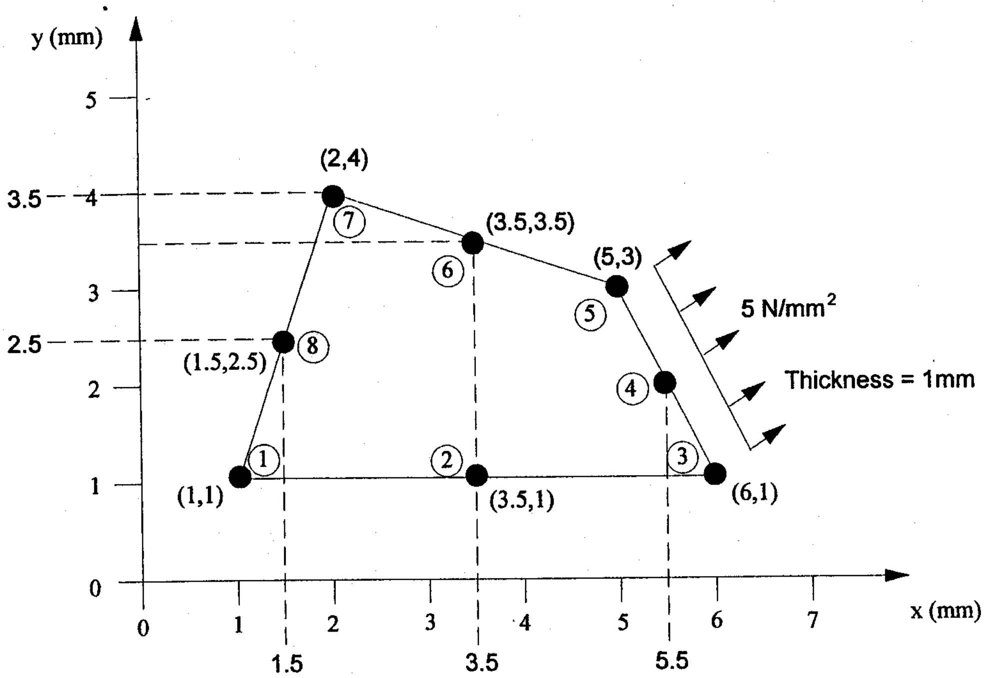

Figure 6(b)

7. The transverse displacement of a simple two-noded beam bending element of length L is written as

$$v(x)=N_1(x)v_1+N_2(x)\theta_1+N_3(x)v_2+N_4(x)\theta_2$$

where v_1 and v_2 are the nodal tranverse displacements and θ_1 and θ_2 are the nodal rotations. The shape functions are given as follows:

$N_1(x) = (1+2x/L)(1-x/L)^2$
$N_2(x) = x(1-x/L)^2$
$N_3(x) = (x/L)^2(3-2x/L)$
$N_4(x) = x(x/L)(-1+x/L)$

Determine the equivalent element nodal load vectors for the following cases:

(i) A distributed pressure load of intensity q *(6)*
(ii) A concentrated load P at the element midpoint *(6)*
(iii) A counter-clockwise couple M applied at the element midpoint. *(8)*