*Ollscoil na hÉireann, Gaillimh*
*National University of Ireland, Galway*

GX 816

## Semester II Examinations, 2002/2003

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|---|---|
| Exam Code(s) | 4BM121, 4BG121, 4BN121 |
| Exam(s) | 4th Mechanical Engineering<br>Mechanical, Biomedical, Electronic |
| Module Code(s) | ME403 |
| Module(s) | Computer Control of Electro-Mechanical |
| Paper No. | 1 |
| Repeat Paper | Special Paper |
| External Examiner(s) | Prof. P.J. Mallon |
| Internal Examiner(s) | Prof. J.F. McNamara<br>J.P. Dunne |

**Instructions:**

Answer 5 questions
All questions marked equally
Attached are the following information

- Table of z transforms
- Table of Equivalent Discrete-time filters for a continuous time filter
- Table of important properties and theorems of the z transforms

| | |
|---|---|
| Duration | 3hrs. |
| No. of Answer books | |

**Requirements**:

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|---|---|
| Handout | |
| MCQ | |
| Statistical Tables | Yes |
| Graph Paper | |
| Log Graph Paper | |
| Other Material | |

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|---|---|
| No. of Pages | 7 |
| Department(s) | Mechanical Engineering |

**1 (a) 10 Marks**

Consider the armature controlled dc servo motor, shown in Fig. 1(a). Derive an equivalent block diagram representation and then derive the transfer function relating the output velocity Ω (s) to the system input voltage V(s). [Note: You may assume that the field current is held constant]

*V - input voltage; $i_a$ - armature current; $L_a$ - armature inductance; $R_a$ - armature resistance; e - back emf; T- torque developed at output; Ω - shaft speed, J (not shown) – moment of inertia of motor and load,; $L_f$ – field inductance; $R_f$ – motor resistance; $i_f$ – field current; $V_{(field)}$ – field voltage*

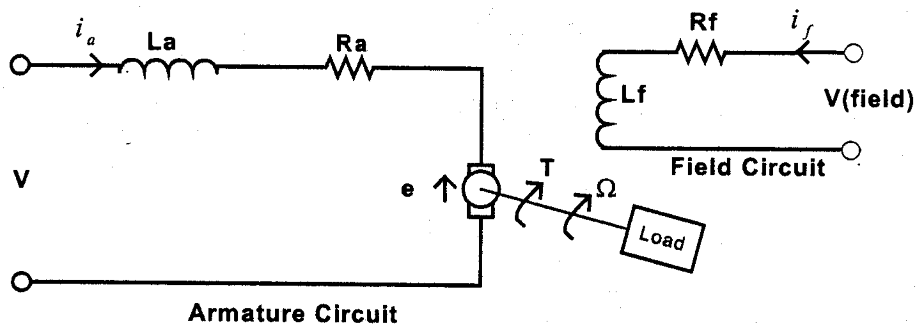


FIGURE 1(a)

**(b) 10 Marks**

Consider the discrete control system illustrated in Fig. 1(b) where the motor is modelled by $G_m(s)$ and the required speed is R(s) and the actual controlled speed is C(s). Derive the pulse transfer function and then determine the response of the system, C(kT) to a step input. Also sketch the form of the output response.

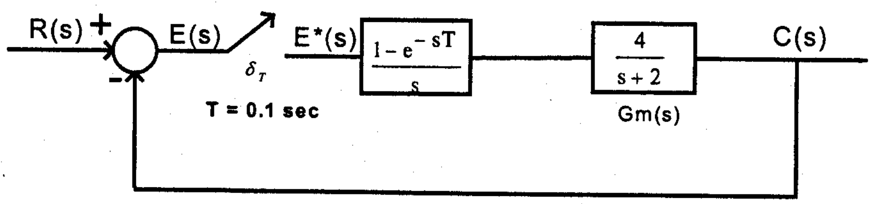


FIGURE 1(b)

**2 (a) 10 Marks**

A Sampler and Zero-Order-Hold is shown in Fig. 2(a), where

$$\overline{e(t)} = e(0)[u(t) - u(t-T)] + e(T)[u(t-T) - u(t-2T)] + e(2T)[(t-2T) - u(t-3T)] + \ldots$$

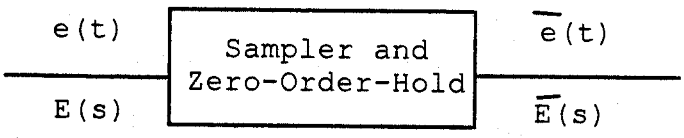


FIGURE 2(a)

Show that the representation in Fig. 2(b) accurately models the input-output relationship of the *Sampler and Zero-Order-Hold*. Explain what is meant by an ideal sampler and why it is also described as an 'impulse sampler'. Derive an expression for E*(s) and e*(t). Explain the significance of the two terms.

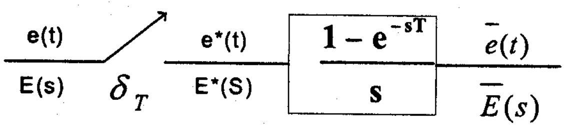


FIGURE 2(b)

**(b) 10 Marks**

Obtain the discrete-time output C(z) of the closed loop control system shown here in Fig. 2(c). Also obtain the continuous time output C(s). Determine if it is possible to get the pulse transfer function; explain your answer.

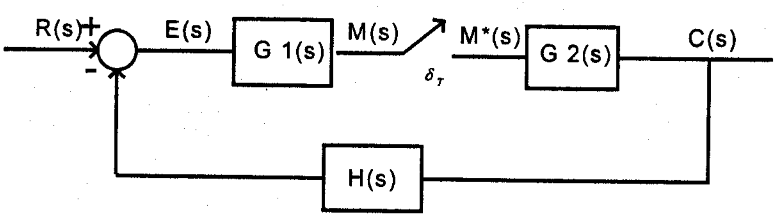


FIGURE 2(c)

3 (a) **10 Marks**

Show that the circuit given in Fig. 3(a) approximates the a Zero-Order-Hold, where m(t) := e(kT). Your answer must outline any assumptions made

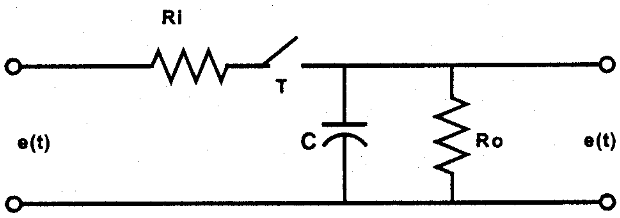


FIGURE 3(a)

(b) **10 Marks**

Explain briefly what is meant by the block diagram realization of controllers. Assume a digital filer is given by the following difference equation shown here. Draw block diagrams for the filter using (1) direct programming and (2) standard programming.

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1(X)$$

4 (a) **10 Marks**

Consider the discrete time control system outlined in Fig. 4(a). Obtain the discrete time output C(z) and the continuous time output C(s) in terms of the input and the transfer function of the blocks.

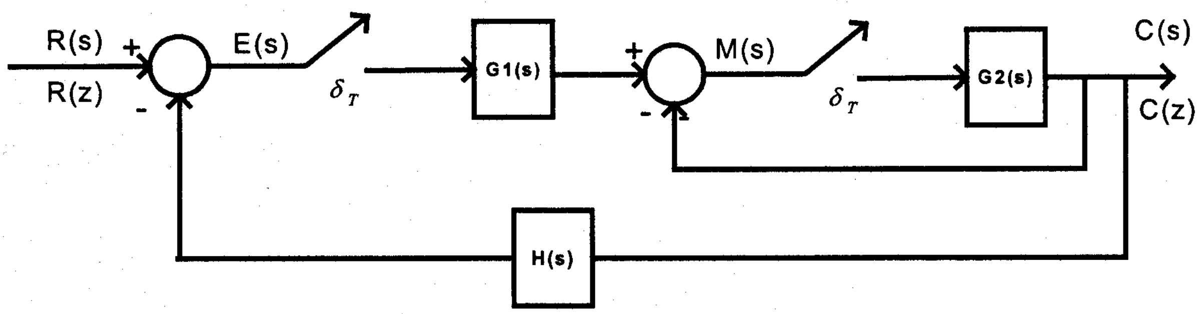


FIGURE 4(a)

(b) **10 Marks**

Determine the step response of the system in Fig. 4(b) . The digital filter, D(z) is described by the difference equation;

$$m(kT) = 2e(kT) - e[(k-1)T]$$

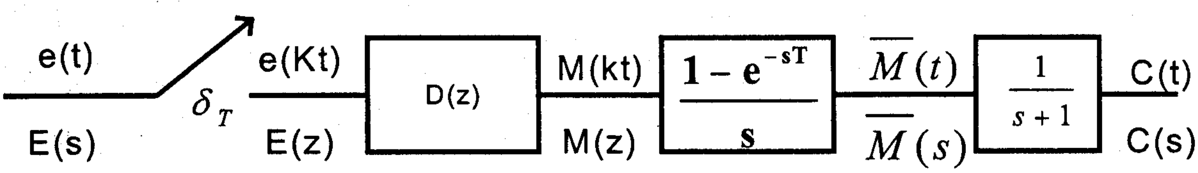


FIGURE 4(b)

**5 (a) 10 Marks**

Consider the block diagram for the continuous-time control system shown in Fig. 5(a). Draw a block diagram of the system if a digital control replaced the continuous controller. Your answer must also include a discussion on the elements, their inter-connection and the different types of signals that would appear through out the system. Discuss the impact of using a piecewise continuous signal, $\bar{u}(t)$ as the controlling signal for the plant.

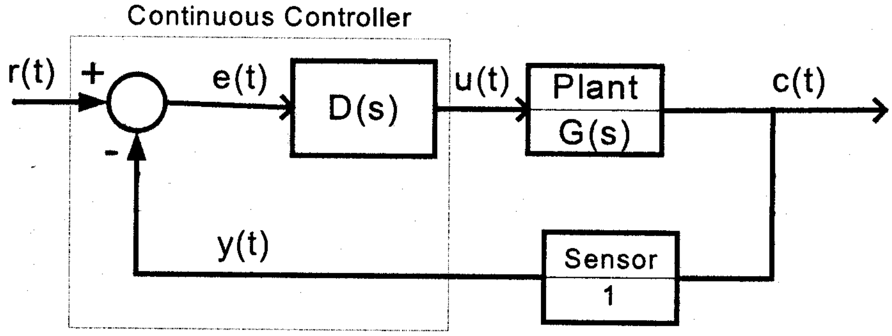


FIGURE 5(a)

**(b) 10 Marks**

Describe Euler's method for approximating differential equations and show how it can be used to realize a difference equation for a lag/lead compensator whose general form is

$$D(s) = \frac{U(s)}{E(s)} = K_0 \frac{s+a}{s+b}$$

In addition, derive the difference equation for P(s) at sampling rates of 20Hz and 40Hz, respectively.

$$P(s) = 70\frac{s+2}{s+10}$$

**6 (a) 10 Marks**

Consider the Radio-Telescope system shown in Fig. 6(a). The diagram on the right is a simplified lumped parameter model suitable for the purposes of this analysis. Derive the transfer function relating actual antenna orientation $\theta_0$ to desired antenna orientation $\theta_i$. Also show how the form of the transfer function relates to the general form of a second-order system and drive expressions for the natural frequency, ω(n) and the damping factor ξ.

*$K_f$ := Opposing frictional torque const.; J := Lumped moment of mass of antenna; $K_s$ :=Lumped torsional stiffness const of structure*

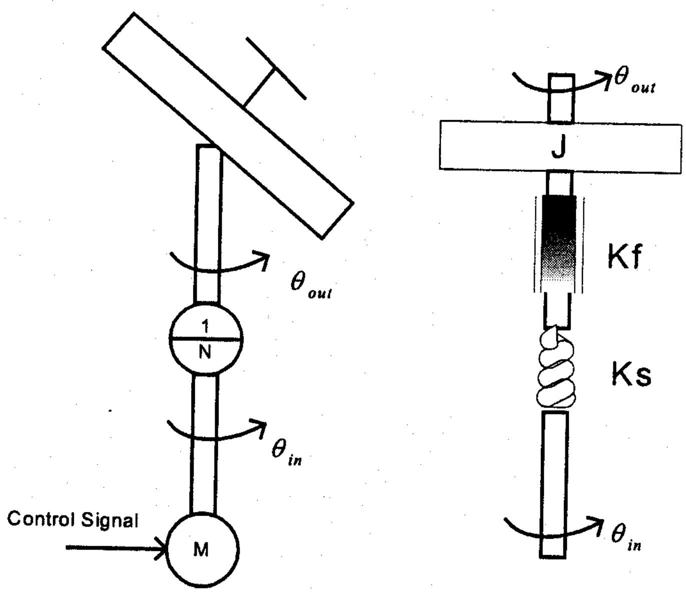


FIGURE 6(a)

**(b) 10 Marks**

Consider the discrete time control of a second-order system as shown in Fig. 6(b). Determine the range of the gain K for stability using the *Jury Stability Criterion.*

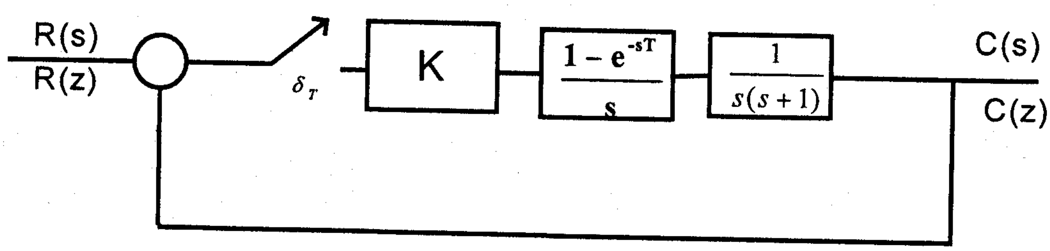


FIGURE 6(b)

7 (a) 7 Marks

Determine an expression for C(z) when the input e(t) is a unit step function, as shown in Fig. 7(a).

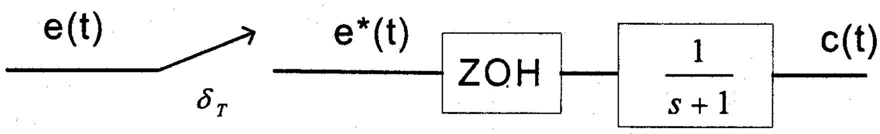


FIGURE 7(a)

**(b) 13 Marks**

Draw the Root-Locus diagram, in the z-plane, for the system shown in Fig. 7(b). The sampling period is T= 1 sec. Your answer must show the value of K at the breakaway points.

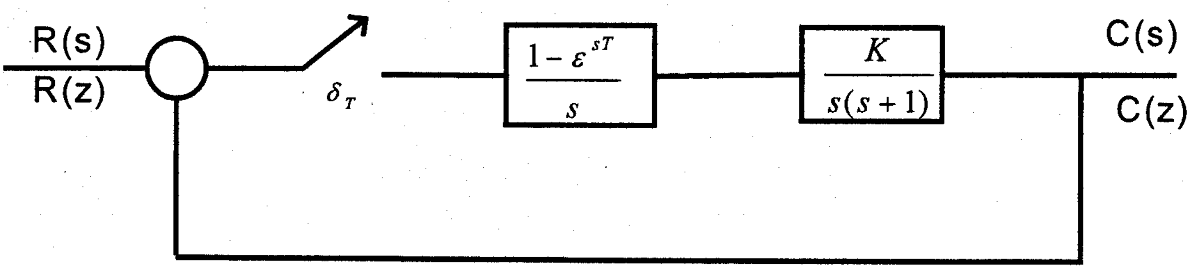


FIGURE 7(b)

**8 (a) 4 Marks**

Briefly discuss the rationale and issues relating to the development of discrete-time equivalents of continuous time filters.

**(b) 8 Marks**

Consider the simple continuous-time (analog) filter shown in Fig. 8(b). The value of the resistance R is 1000 Ω and the capacitor is $100 \times 10^{-6}$ Farads. Derive the transfer function

$$\frac{Y(s)}{X(s)}$$

Obtain the equivalent discrete-time filter using the bi-linear transformation method with frequency prewarping. It is desired to have the same magnitude at $\omega = 10$rad/sec for both the continuous-time and discrete-time filters. Assume the sampling frequency T is given as 0.2 sec, or T =0.2.

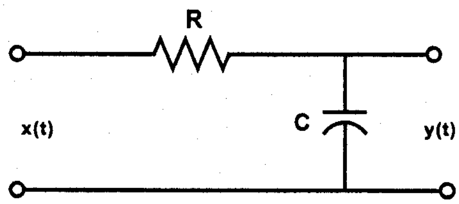


FIGURE 8(b)

**(c) 8 Marks**

Briefly discuss the design of discrete-time control systems based on the frequency response method. Obtain the G(*w*) for the system shown in Fig. 8(c) using the bilinear transform. Verify your answer using

$$\lim_{w \to 0} G(w) = \lim_{s \to 0} G(s)$$

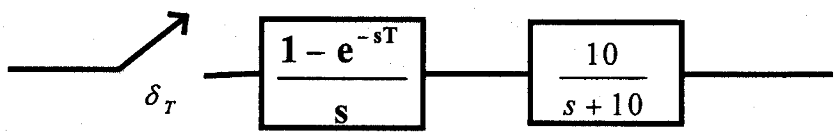


FIGURE 8(c)