OLLSCOIL NA hÉIREANN, GAILLIMH

THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

WINTER EXAMINATIONS 2002

2nd Year Industrial Engineering and Information Systems Examination

HYDRAULICS (EH 204)

Examiners: Professor K. Beven
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Attempt *any* **four** questions

(The Formula Handout provided may be useful in one or more questions)

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1. [Resultant Hydrostatic Thrust on Plane Areas and Centre of Pressure]

(a) *Very briefly*, what do you understand by each of the following **terms**:

Hydrostatic Pressure, **Resultant Hydrostatic Thrust**, and **Centre of Pressure** ?

[1 mark]

(b) Suppose that an **open tank, with vertical side walls, is square in plan,** the side walls being of width B metres. If the depth of a liquid in the tank is H metres, **show that**

(i) the **hydrostatic liquid thrust** (T_{floor}) acting on the **floor of the tank** is

$$T_{floor} = \rho g H B^2$$

where ρ is the liquid density and g the acceleration due to gravity. *[1 mark]*

(ii) the **depth of the 'centre of pressure'**, for the **floor of the tank**, is

$$\bar{h}_{floor} = H$$

[1 mark]

(iii) the **hydrostatic liquid thrust** (T_{wall}) acting on **each side-wall** is

$$T_{wall} = \tfrac{1}{2}\rho g B H^2 = \rho g A \bar{x}$$

where A is the *submerged* plane area and $\bar{x}$ is the depth of the centroid of A. *[2 marks]*

(iv) the **depth of the 'centre of pressure'**, for **each side-wall**, is given by the

$$\bar{h}_{wall} = \tfrac{2}{3}H, \textit{ for the vertical plane walls of constant width } B$$

[3 marks]

(c) If the tank described in part (b) contains a depth H_1 = 3m of **oil**, of density $\rho_{oil} = 900\,kg\,m^{-3}$, and also a depth H_2 = 3 m of **water**, of density $\rho_{water} = 10^3\,kg\,m^{-3}$, the width of each wall of the tank being B = 4 m, using the **pressure diagram, determine**

(i) the **pressure at the depths** 3 m and 6 m, and *also* the **hydrostatic thrust** T_{floor} **acting on the floor of the tank,** *[3 marks]*

(ii) the ***resultant*** **hydrostatic liquid thrust** (T_{wall}) acting on **each side-wall,** *[4 marks]*

(iii) the **depth of the '*centre of pressure*'**, for **each side-wall** of the tank. *[5 marks]*

2. **[Hydrostatic Pressure – Depth Relation and a Differential Manometer]**

(a) Very *briefly*, **distinguish** *clearly* **between each of the following pairs of terms**;

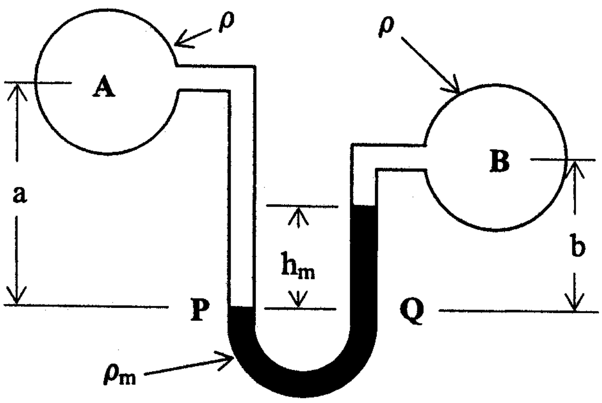
Atmospheric Pressure	and	**Vaporization Pressure,**
Gauge Pressure	and	**Absolute Pressure,**
Liquid Pressure	and	**Pressure Head of Liquid.**

[2 marks]

(b) Show, *from first principles*, that the **gauge pressure at a depth** h below the *free* surface of a liquid *at rest* is given by the equation $P_h = \rho g h$, where ρ is the mass density of the liquid and g is the acceleration due to gravity. *[2 marks]*

(c) Taking the specific gravity (relative density) of mercury (Hg) as s = 13.6, the density of water as $\rho_{water} = 10^3\,kg\,m^{-3}$ and the atmospheric pressure as $P_{atm} = 101.3\,kPa$, express the atmospheric **pressure head** in metres of (i) **mercury** (ii) **water.** *[4 marks]*

(d) Two *horizontal* pipes, carrying the *same* liquid, are connected by a **mercury U-tube manometer**, as illustrated in the accompanying sketch. Derive the relationship between the **liquid pressure head difference** δH between the pipe centre-lines **A** and **B** and the **mercury deflection head** h_m in the U-tube, as



$$\delta H = \frac{P_A - P_B}{\rho g} = h_m\left(\frac{\rho_m}{\rho} - 1\right) + (b - a),$$

where ρ is the density of the liquid in each pipe, ρ_m is the density of mercury in the U-tube and a and b are the potential (elevation) heads of the centre-lines **A** and **B** respectively above the **P - Q** datum shown in the sketch. *[8 marks]*

(e) If, as in your **Hydraulics Laboratory experiment**, the ends of the **differential mercury U-tube manometer** are inserted into **a single horizontal pipe**, rather than the pair of pipes shown in part (d) above, the flowing liquid in the pipe being oil of density $\rho_{oil} = 857.9\,kg\,m^{-3}$ and the specific gravity of the mercury being s = 13.6, **check** if the *estimate* of the **pressure drop** $\delta P = (P_A - P_B) = 10\ kPa$ is consistent with the *measured* mercury head difference of h_m = 0.08 m in the U-tube. *[4 marks]*

3. **[Venturi Meter & Orifice Plate, as Discharge Meters]**

(a) Very *briefly*, **distinguish** *clearly* **between each of the following pairs of terms**;

Real Fluid	and	**Ideal Fluid,**
Steady Flow	and	**Uniform Flow,**
Venturi Meter	and	**Orifice Plate**

[2 marks]

(b) Using the **Bernoulli and Continuity equations**, for **steady flow of an ideal fluid**, **derive the expression**

$$Q_{theoretical} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gH}$$

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for the **theoretical discharge** through a **Venturi Meter** inserted in a pipeline, involving a **gradual contraction and subsequent gradual expansion** across the meter, from the cross-sectional area A_1 of the pipe at the inlet, to A_2 at the throat section, back to A_1 at the outlet of the meter, H being the drop in the piezometric head from the inlet to the throat sections and g the acceleration due to gravity. *[8 marks]*

(c) Explain *very briefly* why the **above expression also holds** for the theoretical discharge through an **Orifice Plate** and *briefly* **compare these two discharge meters**. *[3 mark]*

(d) *Briefly* explain how the theoretical expression in part (b) can be modified, for the flow of a **real (rather than ideal) fluid**, by using a *constant* **Coefficient of Discharge** (C_d), to yield a corresponding expression for the **actual discharge** (Q_{actual}), thereby facilitating the use of the **Venturi Meter and Orifice Plate** as **discharge meters**. *[2 mark]*

(e) Explain *briefly* how, in the **Hydraulics Laboratory**, you verified the *form* of the expression for Q_{actual} and also estimated the C_d value for the **Venturi Meter**. *[5 marks]*

4. [The Hagen-Poiseuille Equation for Laminar Flow in a Single Pipe]

(a) *Show* that the **pipe resistance equation, corresponding to the Hagen-Poiseuille head-loss equation** for **'laminar' flow** of a **'Newtonian' fluid**, is $f = 64/Re$. *[2 marks]*

(b) **Crude oil**, of density $\rho = 860\,\text{kg}\,\text{m}^{-3}$ and kinematic viscosity $\nu = (\mu/\rho) = 18.6 \times 10^{-6}\,\text{m}^2\text{s}^1$, flows out of a **ventilated reservoir** through **a straight horizontal pipe** of length $L = 15\,\text{m}$, diameter $D = 0.01\,\text{m}$ and absolute roughness $k = 0.03 \times 10^{-3}\,\text{m}$. The entry to the pipe is *'sharp-edged'* ($K \approx 0.5$) and a valve ($K = 0.19$) is fitted at mid-length. The **pipe discharges freely into the atmosphere**, at a point located $H = 20\text{m}$ below the oil surface level in the reservoir. **Assuming that the flow in the pipe is *laminar*, check the following calculator results**:

(i) **the discharge in the pipe is $Q = 0.16915 \times 10^{-3}\,\text{m}^3\text{s}^{-1}$.** *[8 marks]*

(ii) **This value of Q is consistent with the assumption of laminar flow.** *[2 marks]*

(iii) **The ratio of shock to friction head losses is only 2.038%.** *[4 marks]*

(iv) **If the fluid was considered *'ideal'*** (i.e. *all* losses neglected), the **% error** ***(over-estimation)*** in the resulting value of Q would be $\delta Q =$ **821.09%**. *[4 marks]*

5. [The Coefficient of Discharge for a sharp-edged Rectangular Weir]

(a) **State** (i) the '**Continuity equation for steady flow**' and *[1 mark]*

(ii) the '**Bernoulli equation for steady flow of an ideal fluid**'. *[1 mark]*

(b) Derive the **Torricelli equation** $V = \sqrt{2gH}$ for the **mean velocity of an ideal fluid** flowing through a **small orifice** of diameter D, located on the side of a tank at a depth H below the free liquid surface, where $H >> D$, and show that the corresponding **theoretical discharge** is given by $Q_{theoretical} = \dfrac{\pi D^2}{4} \times \sqrt{2gH}$. *[4 marks]*

(c) Show that, for an ***ideal* fluid**, the ***theoretical* discharge from a large open reservoir**, over a thin *vertical* **sharp-edged rectangular weir** fitted to its side, has the form

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$$Q_{theoretical} = \frac{2}{3} B \sqrt{2g}\, H^{\frac{3}{2}}$$

where B (in m) is the width of the weir, H (in m) is the difference in elevation between the free horizontal surface level in the reservoir and the crest of the weir, g is the acceleration due to gravity (in m^2s^{-1}) and $Q_{theoretical}$ has the units of m^3s^{-1}. *[6 marks]*

(d) Suppose that the following are a set of **experimental measurements** for a *fully-contracted* **sharp-edged rectangular weir** of width $B = 2.5$ m;

Head H (m)	0.5	1.0	1.5	2.0	2.5	3.0
Discharge Q (m^3s^{-1})	1.626	4.599	8.449	13.009	18.180	23.898

(i) *Explain very briefly* how **the weir Coefficient of Discharge C_d can be got empirically,** from such a **Hydraulics Laboratory** data set. *[4 marks]*

(ii) Using *any pair of values* in the *above* table, *or otherwise*, **do a rough numerical check** to ascertain if the **value $C_d \approx 0.623$ is appropriate for this weir**. *[4 marks]*

6. **[Power of a Jet using the Momentum Equation for Steady Flow]**

(a) **State** the **Momentum Equation** for the **force F** exerted on a *control volume* of liquid, due to a **change in momentum flux**, for a **steady discharge** Q, in terms of the change in average velocities V_1 and V_2 across the length L of the control volume. *[2 marks]*

(b) A ***free* jet of liquid discharges *vertically*** from a nozzle of area a, with mean velocity U, and **impinges normally on a stationary horizontal plate**, located a *short vertical* distance s from the nozzle, as shown in Fig. A of the accompanying sketch. **Show**

(i) that the **velocity** of the jet, ***just* before striking the plate**, is $V = \sqrt{U^2 - 2gs}$ *[3 marks]*

(ii) that the **force F exerted on the plate by the jet of liquid** is given by $F = \rho Q V$, where Q is the discharge and ρ the density of the liquid. *[4 marks]*

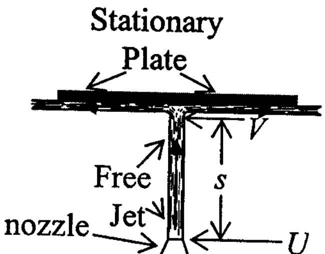

Fig. A

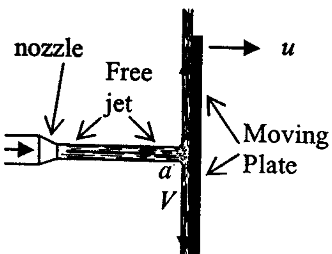

Fig. B

(c) If, instead, the **jet is horizontal** and the **plate vertical**, as shown in Fig. B of the sketch in part (b), and **the plate on which the jet impinges acquires a *horizontal* velocity *u away* from the nozzle.** Assuming that the *jet remains horizontal*, **show** that

(i) the **force F on the moving plate** is given by $F = \rho a(V-u)^2$ *[6 marks]*

(ii) the **power P** expended on the moving plate is given by $P = \rho a u(V-u)^2$. *[5 marks]*

7. (a) In the context of **pipe flow**, explain *very briefly* what do you understand by the following **terms** :

laminar flow	**turbulent flow**	**Reynolds Number (Re)**
friction head loss	and	**Shock (form) head loss**

[2 marks]

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(b) Crude oil, of density ρ = 860 kg m^{-3} and kinematic viscosity $\nu = 18.6 \times 10^{-6} \mathrm{m^2 s^{-1}}$, *flows down* at a constant rate from one large open reservoir to another (entering well *below* its oil surface level) through a **pipeline having the following characteristics:**

Pipe Number	Length L (m)	Diameter D (m)	Absolute roughness k (m)
Pipe No. 1	600.0	0.10	0.03×10^{-3}
Pipe No. 2	500.0	0.20	0.06×10^{-3}

Denoting the sum of the **shock losses in a pipe** by $(\Sigma K)\dfrac{V^2}{2g} = \left[\dfrac{8(\Sigma K)}{\pi^2 g D^4}\right]Q^2$ and. including the shock loss at the sudden expansion, take $(\Sigma K)_1 = 4.6125$ for Pipe No. 1 and take $(\Sigma K)_2 = 6.75$ for Pipe No. 2.

Assuming that the ***steady* difference between the reservoir oil surface levels** is $H_s = 7.09\,\mathrm{m}$, **check *fully* the following computer output solution for the discharge** Q in the pipes:

Pipe No.	Discharge ($\mathrm{m^3 s^{-1}}$)	Velocity ($\mathrm{ms^{-1}}$)	Coefficient of Friction f	Head Losses (m)	Relative Roughness k/D	Reynolds No. Re	Flow Type
1	5.84345×10^{-3}	0.74401	0.04021	6.93703	0.3×10^{-3}	4000	Turbulent (just!)
2	5.84345×10^{-3}	0.18600	0.03200	0.15297	0.3×10^{-3}	2000	Laminar (just!)

where the Reynold's Number is defined as $Re = \dfrac{\rho V D}{\mu} = \dfrac{VD}{\nu} = \dfrac{4Q}{\pi D \nu}$ *[18 marks]*

8. **[Pump-Pipeline System Analysis involving a Constant-Speed Pump]**

(a) In the context of **pump-pipeline system analysis and pump selection**, what (*very briefly!*) do you understand by *each* of the following three terms;
the '**pipe system' curve,** the '**manometric head-discharge' curve** and the '**efficiency-discharge curve**'? *[2 marks]*

(b) A **'constant-speed' pump** (1450 rpm) is installed in a pumping station for the purpose of delivering sewage ***up*** to a settling tank through a μPVC pipeline of diameter D = 0.2m and length L = 2.5 km, the **static head lift involved** being $H_s = 15.0$m. For the discharge Q in *litres per second* and H_P in *metres*, the pump has a **manometric head-discharge relation** $H_P = A_1 + B_1 Q + C_1 Q^2 = 60.3289 - 0.4500Q - 0.0017Q^2$.

The *effective* absolute roughness of the pipe is $k = 0.15 \times 10^{-3}$ m, the kinematic viscosity of the sewage is taken as $\nu = (\mu/\rho) = 1.14 \times 10^{-6}\ \mathrm{m^2 s^{-1}}$ and an allowance is made for **form (i.e. shock) losses** amounting to $(\Sigma K)V^2/2g = 10.0\ V^2/2g$.

The **pump characteristics** and **pipe system curve data,** (the Manometric-Head / Discharge and Efficiency / Discharge data) for the **pump** are provided in **Tables A and B** below and also shown graphically in the accompanying figure.

Table A : The Pump Characteristics (Manometric Head & Efficiency versus Discharge)

Discharge (litres/sec)	1.000	15.250	27.750	31.102	37.581	42.015	52.740	62.50	70.000	77.400	81.000
Pump Head (m) for 1450 rpm	59.8772	53.0710	46.5323	44.6886	41.0162	38.4210	31.8673	25.5633	20.4989	15.3146	12.7252
Pump Efficiency (%) for 1450rpm	3.410	44.353	68.498	73.102	79.760	82.613	83.791	77.825	68.684	55.784	48.114

The System Curve (incorporating the Static Head Lift H_s) Ordinates from Table B overleaf.

System Curve Ordinates (m)	15.0245	18.3753	25.5371	28.0920	33.8644	38.4210	51.4800	64.1027	76.4804	89.0413	96.0859
Discharge (litres/sec)	1.000	15.250	27.750	31.102	37.581	42.015	52.740	62.50	70.000	77.400	81.000

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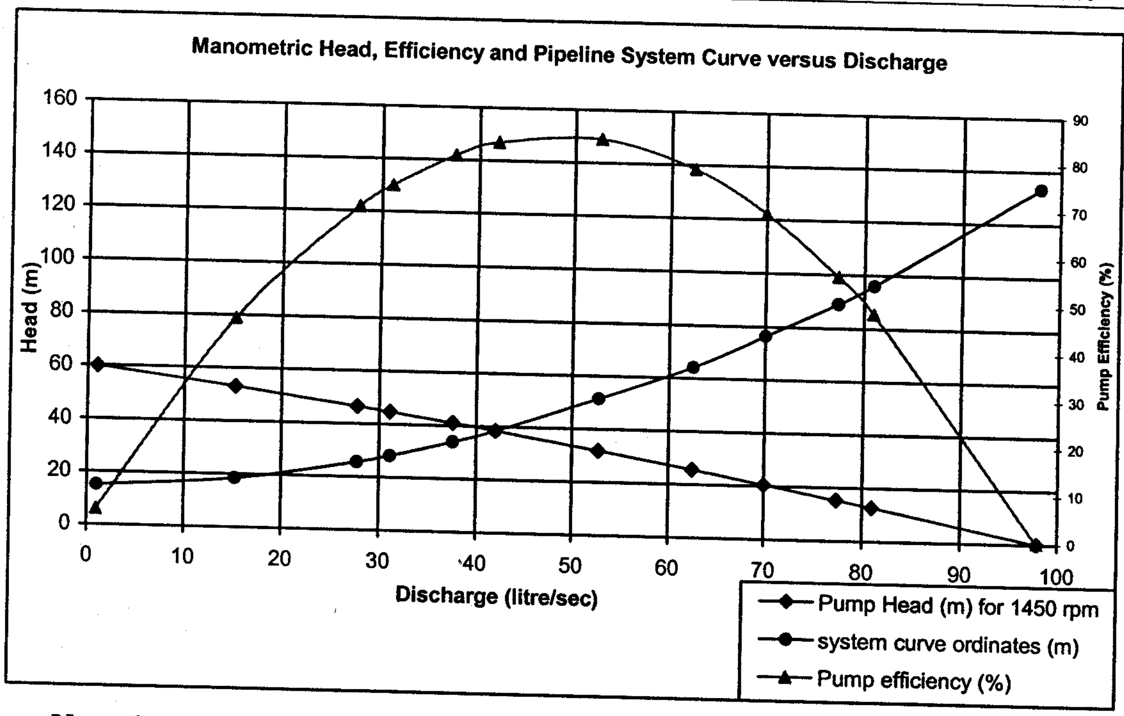
Table B : Calculations for the Pipe System Curve, including the Head Lift H_s = 15.00m:

Discharge (litres/sec) Q	Reynolds Number *(Re)*	Friction Coefficient *(f)*	Head loss (m)		Static Head (m) (H_s)	System (plus Static) head (m)
			Friction Loss (h_f)	Shock Loss (h_{sh})		
1.000	5584	0.03717	0.0240	0.0005	15.00	15.0245
15.250	85162	0.02168	3.2552	0.1201	15.00	18.3753
27.750	154967	0.02040	10.1394	0.3977	15.00	25.5371
31.102	173685	0.02017	12.5925	0.4995	15.00	28.0920
37.581	209869	0.01989	18.1350	0.7294	15.00	33.8644
42.015	234630	0.01975	22.5095	0.9116	15.00	38.4211
52.740	294520	0.01952	35.0436	1.4364	15.00	51.4800
62.500	349024	0.01867	47.0854	2.0173	15.00	64.1027
70.000	390907	0.01864	58.9499	2.5304	15.00	76.4804
77.400	432231	0.01835	70.9476	3.0937	15.00	89.0413
81.000	452335	0.01835	77.6977	3.3882	15.00	96.0859
97.875	546572	0.01834	113.4270	4.9470	15.00	133.3740



Note that a *separate enlarged (full-page) version* of this figure for Q.8 is provided with this Examination Paper **for submission with your Examination Answer Book.**

Using the <u>**enlarged version**</u> **of the figure, (and Tables A and B),**

(i) <u>**Indicate *graphically* the discharge *Q**</u> in the pipe, <u>**the pump head H_P *delivered***</u> to the pipeline and the corresponding <u>**efficiency η of the pump**</u> *[3 marks]*

(ii) *Estimate* the **Hydropower (in kW) delivered to the pipeline by the pump.** *[2 marks]*

(iii) *Estimate* the **Mechanical Input Power to the pump** (in kW). *[2 marks]*

(iv) If the **motor driving the pump** is operating at an efficiency of η_{motor} = 65%, estimate the **Electrical Input Power (in kW) to the motor.** *[2 marks]*

(c) By interpolation of the tables provided (*if necessary*), **check analytically your solution for the discharge Q** in the pipe. *[9 marks]*

END