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NATIONAL UNIVERSITY OF IRELAND
GALWAY

SEMESTER 1 EXAMINATIONS 2002-2003

3rd year B.Sc.Unit EP325: Quantum Physics

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Time allowed : TWO hours.	Answer THREE questions

Note: A list of some formulae is given at the end of this paper.

Q.1 Explain what is meant by the term *blackbody*, and suggest how a near perfect blackbody can be produced for experimental purposes. *[1 mark]*

Derive an expression for the number of allowed modes of electromagnetic waves in a cubic cavity of side length L. Hence write down the Rayleigh-Jeans formula for the energy density of a blackbody as a function of frequency. *[5 marks]*

How does the Rayleigh-Jeans formula compare with the observed experimental data? *[1 mark]*

Briefly explain how Planck derived a correct formula for the *blackbody* distribution *[1 mark]*

Explain what is meant by *Wien's Displacement Law*. Briefly mention how you might use it to measure the surface temperature of a star. *[2 marks]*

Q.2 Show that when the Time Dependent Schrödinger Equation is applied to a system, in which the potential is constant with time, it is possible to find solutions of the form: $\Psi(x,t) = \psi(x)\phi(t)$.
Determine $\phi(t)$ and explain how $\psi(x)$ may be obtained *[3 marks]*

Solve the time independent Schrödinger Equation for a particle of mass m in a potential well defined by:

$V(x) = 0$ for $|x| \le a$,
$V(x) = \infty$ for $|x| > a$. *[4 marks]*

Derive an expression for the allowed particle energies. *[2 marks]*

Sketch the waveform for the lowest energy bound solution in the case where the potential well is of *finite* depth. *[1 mark]*

Q.3 Show that if Ψ_1 and Ψ_2 are solutions to the Time Dependent Schrödinger Equation then so also is a linear combination of Ψ_1 and Ψ_2. *[2 marks]*

Show that non-degenerate solutions to the Time Independent Schrödinger Equation are ortho-normal. *[4 marks]*

The state function of a system can be expressed as a linear combination of energy eigenfunctions as follows:

$$\Psi(x,t) \;=\; \sum_n c_n\, \psi_n(x) e^{-\frac{E_n}{\hbar}t}$$

Calculate the expectation value for energy <E>. Hence, comment briefly on the physical interpretation of c_n. *[4 marks]*

Q.4 State the continuity properties required of wavefunctions that are physically acceptable solutions to the Schrödinger Equation. *[1 mark]*

A potential barrier is defined in one dimension as follows

$x < 0$	$V(x) = 0$
$0 \le x \le L$	$V(x) = V_0$
$x > L$	$V(x) = 0$

Write down the general solution to the Schrödinger equation in each of the three regions. *[4 marks]*

A particle with energy $E = V_0 / 2$ approaches the barrier. Derive an expression for the probability that the particle will be transmitted across the barrier. *[5 marks]*

Q.5 Answer *TWO* of the following.

(a) Briefly explain was is meant by the commutator of two operators. What is the physical significance of the commutator? Determine the commutator of the one-dimensional position and momentum operators. *[5 marks]*

(b) Derive an expression for the ground state electron wave function in a Hydrogen atom. Sketch how the position probability density varies as a function of r. *[5 marks]*

(c) Given the probability relationship for Fermions

$$P_F(n_s) \;=\; \prod_s \frac{g_s!}{n_s!\,(g_s - n_s)!}$$

where the symbols have their usual meanings, derive the Fermi-Dirac distribution function. *[5 marks]*

Schrödinger Equation: $$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x,t)\right]\Psi(x,t)$$

Taylor's Series: $$f(a+h) = f(a)+hf'(a)+\frac{h^2}{2!}f''(a)+\frac{h^3}{3!}f'''(a)+\ldots$$

Binomial Expansion: $$(x+y)^n = x^n+nx^{n-1}y+\frac{n(n-1)}{2!}x^{n-2}y^2+\ldots$$

Normal Distribution: $$f(x) = \frac{1}{\sigma\sqrt{2}\,\pi}\exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$$

Standard Deviation: $$\sigma = \sqrt{\frac{1}{N}\sum_i^N\left(x_i-\mu\right)^2}$$

Schwartz Inequality: $$\int f^*f\,dx\int g^*g\,dx \geq \frac{1}{4}\left[\int(f^*g+g^*f)\,dx\right]^2$$

Stirling's Approximation: $$\ln n! \approx n\ln n-n$$

$$\nabla^2\Psi=\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2} = \frac{1}{r^2\,Sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\,Sin\theta\frac{\partial\Psi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(Sin\theta\frac{\partial\Psi}{\partial\theta}\right)+\frac{\partial}{\partial\phi}\left(\frac{1}{Sin\theta}\frac{\partial\Psi}{\partial\phi}\right)\right]$$

$$\frac{d}{dx}\ln f(x) = \frac{1}{f(x)}\frac{d}{dx}f(x)$$

$$\int u\,dv = uv-\int v\,du$$

$$\int_0^\infty x\,e^{-x}\,dx = 1$$

$$\int_0^\infty\frac{x^3\,dx}{e^x-1} = \frac{\pi^4}{15}$$

$$I_0 = \int_0^\infty e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$I_1 = \int_0^\infty x\,e^{-\alpha x^2}\,dx = \frac{1}{2\alpha}$$

$$I_n = \int_0^\infty x^n\,e^{-\alpha x^2}\,dx\,,\quad I_{n+2} = -\frac{dI_n}{d\alpha}$$

$$Sin(A+B) = Sin\,A\,Cos\,B+Cos\,A\,Sin\,B$$

$$Cos^2\,A = \frac{1+Cos\,2A}{2}$$