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GAILLIMH

NATIONAL UNIVERSITY OF IRELAND
GALWAY

SEMESTER 1 EXAMINATIONS 2002-2003

3rd year B.Sc.Unit EP311: Light and Electromagnetism

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Time allowed : TWO hours.	Answer THREE questions

The numerical values of some physical quantities are given at the end of the paper.

A formula sheet is also appended.

Q.1 Derive and discuss Maxwell's equations in integral form, as listed in the formula sheet. Discuss the physics involved in each case. Explain the basis on which Maxwell arrived at the concept of displacement current. [10 marks]

Q.2 Explain the concept of the Poynting vector. Derive an expression for it in terms of the electric and magnetic fields of an electromagnetic wave.[4 marks]

On the basis that the electric and magnetic fields of an electromagnetic wave can be represented by harmonic waves, show that the expression

$$I = \frac{1}{2\mu_o c} \left|\vec{E}_o\right|^2$$

gives the irradiance of an electromagnetic wave. [3 marks]

A light wave, in free space, has an irradiance of 50 Wm^{-2}. Calculate

(i) the amplitude of its electric field,
(ii) the amplitude of its magnetic field, and
(iii) the magnitude of the Poynting vector. [3 marks]

Q.3 Write down an expression for the electric field component (as $\nabla^2 \vec{E}$) of an electromagnetic wave in a conductor. On the assumption that this equation has a harmonic solution of the form,

$$\vec{E} = \vec{E}_o \, e^{i\omega(t-y/v)}$$

derive an expression for the complex refractive index of metals. Show that when $\sigma >> \varepsilon \, \omega$, both the real and imaginary parts of the refractive index are given by

$$n_R \;=\; n_I \;=\; c\sqrt{\frac{\mu\sigma}{2\omega}}$$

Explain the concept of skin depth for metals and derive an expression for it.

[7 marks]

In the case of copper ($\sigma = 5.88 \times 10^{-7}$S m^{-1}), find the values of the real and imaginary parts of the complex refractive index, for electromagnetic radiation of wavelength $\lambda = 600$ nm. Calculate the value of the skin depth at the same wavelength.

[3 marks]

Q.4 Derive an expression for the resultant wave when two waves of the same amplitude, but different frequencies,

$$E_1 = E_{01} \text{ Cos } (k_1 x - \omega_1 t)$$

and

$$E_2 = E_{01} \text{ Cos } (k_2 x - \omega_2 t)$$

overlap in the same region of space, at the same time.

Discuss the nature of the resultant wave, explaining the phenomenon of beats.

[7 marks]

Define phase velocity and group velocity and derive expressions for both in the case of the above resultant wave.

You may find the following trigonometric relationships useful:

$$\text{Cos}\,A + \text{Cos}\,B \;=\; 2\,\text{Cos}\left(\frac{A+B}{2}\right)\text{Cos}\left(\frac{A-B}{2}\right)$$

$$\text{Cos}^2 A = \left(\frac{1+\text{Cos}\,2A}{2}\right)$$

[3 marks]

Q.5 Answer both (a) and (b)

(a) Show clearly how light may be polarised by reflection from the surface of a dielectric. Derive an expression for Brewster's angle, the angle of reflection at which the reflected light will be totally polarised. [5 marks]

(b) Discuss briefly the general expression for elliptical polarisation which is given in the formula sheet, explaining what each of the symbols represents. Using this equation, show how linear polarisation and circular polarisation are special cases of elliptical polarisation. [5 marks]

Numerical values of some physical quantities

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
Speed of light in free space, $c = 3.00 \times 10^{8}$ m s^{-1}

Appendix: EP311 - Light and Electromagnetism - Formula Sheet

3-d differential wave equation (cartesian coords.) $\nabla^2 \psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$

Vector calculus relationship $\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A}$

Maxwell's Equations - Integral form

$$\oint_C \vec{E}.d\vec{l} = -\iint_A \frac{\partial\vec{B}}{\partial t}.d\vec{s}$$

$$\oint_C \vec{B}.d\vec{l} = \mu\iint_A (\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}).d\vec{s}$$

$$\oiint \vec{E}.d\vec{s} = \frac{1}{\varepsilon}\iiint_V \rho\, dV$$

$$\oiint \vec{B}.d\vec{s} = 0$$

Maxwell's Equations - Differential form

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu(\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t})$$

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon}$$

$$\vec{\nabla}.\vec{B} = 0$$

Electromagnetic waves

$$\nabla^2\vec{E} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} + \mu\sigma\frac{\partial\vec{E}}{\partial t} + \frac{1}{\varepsilon}\vec{\nabla}\rho$$

$$\nabla^2\vec{B} = \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} + \mu\sigma\frac{\partial\vec{B}}{\partial t}$$

Energy density $u_E = \frac{1}{2}\varepsilon\left|\vec{E}\right|^2$, $u_B = \frac{1}{2\mu}\left|\vec{B}\right|^2$

Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

Intensity of radiation from a dipole $I = \frac{p_0^2\,\omega^4}{32\pi^2\,\varepsilon_0\,c^3}\,\frac{\sin^2\theta}{r^2}$

Refractive index of a metal $n^2 = c^2\left(\mu\varepsilon - i\frac{\mu\sigma}{\omega}\right)$

Phase velocity $v = -\frac{(\partial\varphi/\partial t)_x}{(\partial\varphi/\partial x)_t}$

Superposition of waves $$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_1 - \alpha_2)$$

Fresnel Coefficients

$$r_{perp} = \left(\frac{E_{0r}}{E_{0i}}\right)_{perp} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{-\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$t_{perp} = \left(\frac{E_{0t}}{E_{0i}}\right)_{perp} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)}$$

$$r_{parl} = \left(\frac{E_{0r}}{E_{0i}}\right)_{parl} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$t_{parl} = \left(\frac{E_{0t}}{E_{0i}}\right)_{parl} = \frac{2n_i\cos\theta_i}{n_t\cos\theta_i + n_i\cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

Elliptical polarization $$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\varepsilon = \sin^2\varepsilon$$

Young's slits (ideal case) $$I = 4I_0\cos^2\frac{\delta}{2}\ ,\quad \delta = ka\sin\theta$$

Thin film interference $$\Lambda = 2n_f\, d\cos\theta_t$$

Michelson Interfermeter $$m\lambda = 2d\cos\theta_m\ ,\quad \theta_p = \sqrt{\frac{p\lambda}{d}}$$

Multiple beam interference (thin film)

$$I_r = I_i\,\frac{F\sin^2\frac{\delta}{2}}{1 + F\sin^2\frac{\delta}{2}}\ ,\quad F = \left(\frac{2r}{1-r^2}\right)^2$$

$$I_t = I_i\,\frac{1}{1 + F\sin^2\frac{\delta}{2}}$$

Single slit diffraction $$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2\ ,\quad \beta = \frac{kb}{2}\sin\theta$$

N harmonic oscillators (ideal case) $$I = I_0\,\frac{\sin^2\frac{N\delta}{2}}{\sin^2\frac{\delta}{2}}\ ,\quad \delta = ka\sin\theta$$

Maxwell-Boltzmann distribution $$\frac{N_j}{N_i} = e^{-(E_j - E_i)/kT}$$

Blackbody energy density / Einstein coefficients $$u_\nu = \frac{8\pi h\nu^3}{c^2}\frac{1}{(e^{h\nu/kT} - 1)} = \frac{A_{ji}}{B_{ji}}\frac{1}{(e^{h\nu/kT} - 1)}$$