Ollscoil na hÉireann, Gaillimh GX 909
National University of Ireland, Galway

Semester II Examinations, 2002/2003

Exam Code(s) 3BS1, 3BS9, 3EL1, 3EL2, 3PT1, 3PT2

Exam(s) 3rd Science

Module Code(s) EP324

Module(s) EP324: Signal Analysis

Paper No.
Repeat Paper Special Paper

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Internal Examiner(s) Professor S. G. Jennings
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Instructions: Answer THREE questions.

Duration 2 hrs
No. of Answer Books 1

Requirements:
Handout
MCQ
Statistical Tables
Graph Paper
Log Graph Paper
Other Material

No. of Pages 7
Department(s) Experimental Physics

EP324: Signal Analysis

Answer THREE questions. Time allowed: TWO hours.

NB: Each question on this paper carries 33 marks.

Q.1 (a) Identify the following two equations and briefly explain their significance. Define and relate the terms T and ω_o used in the equations.

$$f(t) = \sum_{n=-\infty}^{n=\infty} c_n \exp(jn\omega_0 t) \qquad c_n = \frac{1}{T}\int_T f(t)\exp(-jn\omega_0 t)\,dt$$

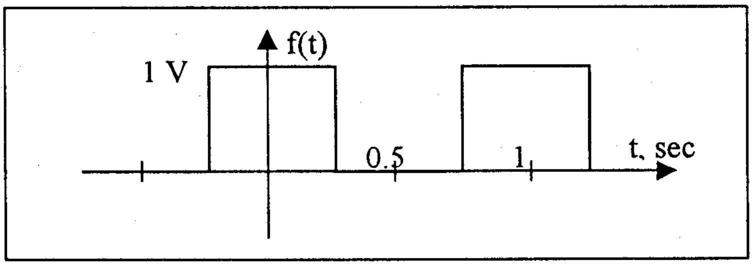
[9 marks]

(b) Prove that the Complex Fourier Series, for the 1 V amplitude, 1 second period, square wave f(t) shown opposite, is given for n = 0 by

$c_0 = 0.5,$

and for n > 0 by

$c_n = \sin(n\pi/2)/(n\pi).$



[10 marks]

(c) Write down a formula for all the c_n ($n \geq 0$) in terms of the *sinc* function, and hence or otherwise make an approximate sketch of the power spectrum of f(t). Calculate the total power, P, in f(t), and also the fraction of P contained in the DC plus first harmonic terms in the Fourier Series.

[14 marks]

Q.2 (a) What essential property of a time function determines whether it will possess a line spectrum or a continuous spectrum? Explain fully what is meant by both of these terms.

[9 marks]

(b) Draw an accurate sketch of the single pulse specified by the conditions below and determine the frequency spectrum for this single pulse

$v = 0$	$\lvert t \rvert > \tau$
$v = V_o(1 + t/\tau)$	$-\tau < t \leq 0$
$v = V_o(1 - t/\tau)$	$0 < t < \tau$

where v represents amplitude, V_o represents peak amplitude, t represents time and τ is a constant.

[16 marks]

(c) Sketch the spectrum obtained and briefly discuss its properties.

[8 marks]

Q.3 Answer (a), (b) *AND* (c).

(a) Explain what is meant by *Linear Time Invariant* (LTI) systems, and briefly state their most important properties. Define the Impulse Response h(t) for such a system, and *explain in detail* how a knowledge of h(t) theoretically allows the calculation of the system output y(t) for any input x(t). What is the usual practical difficulty with this method and how is it overcome?

[11 marks]

(b) Give a complete derivation of f(t)*g(t) for f(t) = 2{exp(-4t)}u(t) and g(t) = 5{cos(3t)}u(t), where the asterisk indicates convolution.

[11 marks]

(c) Find the inverse Laplace transform of

$$F(s) = \frac{2s^2 - 4}{(s-2)(s+1)(s-3)}$$

Give detailed labelled sketches in CT (Continuous Time) of each of the components in the answer.

[11 marks]

Q.4 Answer (a), (b), (c) *AND* (d).

(a) Explain what the process of *discretization* of a CT (Continuous Time) system to a DT (Discrete Time) equivalent is. State the fundamental problems inherent in this process.

[8 marks]

(b) A CT system is described by the differential equation $\frac{dy}{dt} + 3y = x$, where y(t) is the system output and x(t) is its input. Define and use the *Forward Euler Algorithm* to derive the discretization of this equation, for an arbitrary sampling time T, and show the resulting DT difference equation in standard form.

[9 marks]

(c) For T = 0.25 second, calculate the numerical value of the first 3 terms in the DT step response, $y_u[n]$.

[8 marks]

(d) Using the discretization concept, draw an accurate sketch of the convolution of the two functions f(t) and g(t) as described below.

f(t) = 2 for $-1 \le t < 0$ and f(t) = 1 for $0 \le t < 1$

g(t) = 1 for $-1 \le t < 1$

f(t) and g(t) are equal to zero outside these boundaries.

[8 marks]

Q.5 **Answer any *TWO* of the following.**

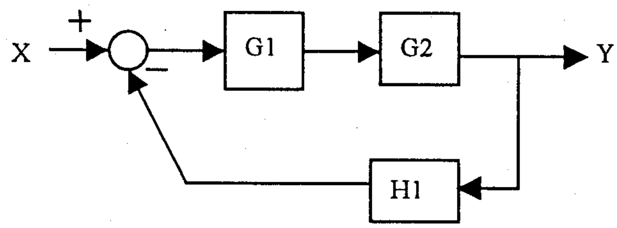
(a) Give an account of Butterworth Low Pass Filters (BW LPF's) of order N, describing their frequency response functions, and stating how their pole positions are determined. Find the pole positions for a 2^{nd} order filter.

Give a circuit diagram (e.g., for the Sallen-Key circuit, or any other) which can implement a 2^{nd} order BW LPF.

[16½ marks]

(b) State the general formula for Mason's Rule, for the transfer function of a system represented in block diagram form. Under what conditions is this formula simplified, and how?

Find the overall transfer function H(s) of the system shown opposite, where G1 = k (k is a variable gain), G2 = 1/(s+1), and H1 = 1/s.



Find the numerical value of k for which the system is a resonant circuit with resonant frequency = 10 rad/sec, and find its corresponding Q value.

[16½ marks]

(c) Describe the general nature of an *underdamped 2-pole system*. Define the system *Q-value*, *damping factor*, *natural frequency* and *damped frequency*, relating them to terms in the general system function H(s). Sketch the approximate nature of the frequency response, and also the impulse response, for such a system.

[16½ marks]