GX 916

Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

Semester II Examinations, 2002/2003

Exam Code(s)	4BS2
	4EL3
Exam(s)	4^{th} Science
Module Code(s)	4BS2-EP447, 4EL3-EP441-6
Module(s)	EP435: Electromagnetism and Special Relativity
Paper No.	
Repeat Paper	Special Paper
External Examiner(s)	Prof. E. Kennedy
Internal Examiner(s)	Prof. S.G. Jennings

Instructions: Answer THREE Questions

Duration 2 Hours
No. of Answer Books

Requirements:
Handout
Statistical Tables
Graph Paper
Log Graph Paper

No. of Pages 4

Department Experimental Physics

EP435 Electromagnetism and Special Relativity

Q.1 Explain what is meant by the following terms in relation to a dielectric material:
Polarization **P**, electric susceptibility χ_e, bound charge, relative permittivity.
[2 marks]

Show that the electrostatic potential $\varphi(\mathbf{r})$ due to polarized material at an external point, **r**, is

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o}\int_{S^1}\frac{\mathbf{P}(\mathbf{r}^1).dS^1}{|\mathbf{r}-\mathbf{r}^1|} - \int_{V^1}\frac{\nabla.\mathbf{P}(\mathbf{r}^1)dV^1}{|\mathbf{r}-\mathbf{r}^1|}$$

and interpret the result. [2½ marks]

How is the bound charge density ρ_b related to the free charge density ρ_f?
[2 marks]

The annular region between the inner conductor, of radius 0.5 mm, and the outer conductor, of radius 2.5 mm of a coaxial cable of circular cross-section is filled with a uniform dielectric polythene material of relative permittivity 2.3. Obtain the capacitance per unit length of the coaxial cable.

$\varepsilon_o = 8.854 \times 10^{-12}$ Fm^{-1} [3½ marks]

Q.2 The magnetic field **B**(**r**) can be expressed by the Biot-Savart law as:

$$\mathbf{B}(\mathbf{r}^1) = \frac{\mu_o}{4\pi}\int_{V^1}\frac{\mathbf{J}(\mathbf{r}^1) \times \mathbf{r}-\mathbf{r}^1 \, dV^1}{|\mathbf{r}-\mathbf{r}^1|^3}$$

[3 marks]

From this, show that $\mathbf{B} = \nabla \times \mathbf{A}$, where **A** is the magnetic vector potential.

Hence show that $\nabla.\mathbf{B} = 0$ [1 mark]

Using the Lorentz condition

$$\nabla . \mathbf{B} = -\varepsilon_o\mu_o\frac{\partial\phi}{\partial t}$$

show that

$$\nabla \times \mathbf{B} = \mu_o(\mathbf{J} + \varepsilon_o\frac{\partial \mathbf{E}}{\partial t})$$

[3½ marks]

Interpret the displacement current density $\varepsilon_o\frac{\partial \mathbf{E}}{\partial t}$ and discuss its magnitude relative to the free current density **J**. [2½ marks]

Q.3 Discuss briefly the contributors to the total current density $\mathbf{J}_{total}$. [2 marks]

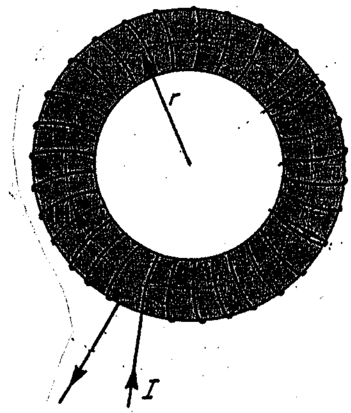
Derive the relationship between the magnetic field intensity $\vec{H}$, magnetic field **B** and magnetization **M**:

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_o} - \mathbf{M}$$

[3 marks]

Show how this relation leads to expressions for sources of current and charge for $\mathbf{H}_{free\ current}$ and $\mathbf{H}_{pole}$ [1½ marks]

Consider a toroidal sample of magnetic material overwound uniformly with a total of N_t turns of wire on the toroid carrying a current I as shown below. Obtain the expression for the magnetic field at a distance r from the centre of the toroid. [3½ marks]



Q.4 Show that the electric field **E** wave and the magnetic field **B** wave travel perpendicular to each other in phase in a non-conducting medium. [3½ marks]

Show for the general case that: $\mathbf{B} = \dfrac{\mathbf{k} \times \mathbf{E}}{\omega}$ [1½ marks]

where **k** is the propagation vector and ω is the angular frequency.

Derive the wave equation for **E** in a conducting medium and briefly discuss the relation between **E** and **B** in such a medium. [5 marks]

Q.5 Write down the Lorentz transformation between two coordinate frames moving relative to each other with constant velocity, v, in the x direction. [1 mark]

Use this to explain the phenomena of

(a) time dilation and [2 marks]
(b) the Fitzgerald-Lorentz contraction [2 marks]

Define the electromagnetic four-vectors $\mathbf{J}_\mu$ and $\mathbf{A}_\mu$. [2 marks]

Show through the use of the Lorentz gauge, that these four vectors are related by

$$\Box^2 \mathbf{A}_\mu = -\mu_o \mathbf{J}_\mu$$

[3 marks]

<u>Some vector identities:</u>

$$\nabla . (\phi \mathbf{V}) = \phi \nabla . \mathbf{V} + \mathbf{V} . \nabla \phi$$

$$\nabla \times (\mathbf{G}\phi) = \nabla\phi \times \mathbf{G} + (\nabla \times \mathbf{G})\phi$$

$$\nabla \times \nabla \times \mathbf{F} \qquad \nabla(\nabla . \mathbf{F}) - \nabla^2 \mathbf{F}$$

$$\nabla^2 \frac{1}{|\mathbf{r} - \mathbf{r}^1|} = -4\pi\delta(\mathbf{r} - \mathbf{r}^1)$$

$$\oint_S \nabla \times \mathbf{F} . d\mathbf{S} = \oint_C \mathbf{F} . d\ell$$