Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX 1608

Semester II Examinations, 2002/2003

Exam Code(s)	2IF1, 3EL2
Exam(s)	IF1 B.Sc. (Information Technology)
	EL2 B.Sc. Degree (Applied Physics and Electronics) (Hons.)
Module Code(s)	CT214
Module(s)	Logical Foundations Of Computing
Paper No.	1
Repeat Paper	
Special Paper	
External Examiner(s)	Dr. Dave Johnson, Professor P. Nixon
Internal Examiner(s)	Professor G. Lyons
	Dr. N. Madden
	Dr. M. Mc Gettrick

Instructions

Answer 4 questions, two from each section.
All questions will be marked equally.
Please use separate answer books for sections A and B

Duration	3hrs
No. of Answer Books	2

Requirements

Handout	
MCQ	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	

No. of Pages	4
Department(s)	

Please use separate answer books for Sections A and B.

SECTION A
Answer Only 2 Questions

A1. (i) Give the logic tables for the NOT ($\neg$), AND ($\wedge$) and OR ($\vee$) gates.
Show that $\neg(x \wedge y)$ is logically equivalent to $(\neg x) \vee (\neg y)$.
Is the compound proposition $\neg(x \to y)$ logically equivalent to $x \wedge (\neg y)$?

(ii) Construct the logic table for the Boolean expression

$$(x \vee y \vee z) \wedge (x \to y) \wedge (y \to \neg z).$$

Write down the corresponding disjunctive normal form (DNF).

(iii) Explain what is meant by saying that a proposition is a **tautology**, or that it is a **contradiction.**

For each of the following compound propositions, determine with justification if it is a tautology, a contradiction, or neither:

(a) $x \to x$.

(b) $\neg x \to x$.

(c) $\neg(x \wedge \neg y) \to (x \to y)$.

A2. (i) What is the definition of a **valid argument**?
For each of the following arguments, determine if it is valid:

(a) $p \to q, \neg q \therefore \neg p$.

(b) $q, \neg p \to \neg q \therefore p$.

(c) $p \vee q, q \to s, \neg(p \vee t) \therefore s \wedge \neg t$

(ii) When is an instance of a valid argument **sound**?
When is an instance of a valid argument **unsound**?
Give an example of each case based on an argument from part (i).

(iii) A lecturer makes the following claim:

"*If a student attends all the lectures then (s)he will pass the exam*".

Six students (A–F) sat the examination. Anne went to all the lectures, as did Connor. Bob did not go to any. Also Dana failed, while Eamon and Fergal both passed.
Determine, with justification, which students should be questioned to see if the lecturer's claim is correct.

A3. (i) When is a collection of compound propositions *inconsistent*?
Describe the **tableau** method for testing a collection of well-formed formulae (compound propositions) for (in)consistency.

(ii) Use the tableau method to show that the following collection of well-formed formulae (WFFs) is consistent:

$$\{p \to (q \vee r), \quad q \to p, \quad \neg r\}$$

Read off from the tableau an assignment of values to p, q and r which makes all three WFFs take the value 1.

(iii) How can the tableau method be use to check if an argument is valid?
Use the tableau method to show that the following argument is valid:

$$p \vee q, \quad p \to r, \quad q \to r \quad \therefore \quad r.$$

SECTION B
Answer Only 2 Questions

B1. (i) Explain (with an example) the following properties as they apply to the $\wedge$ and $\vee$ logical connectives: Commutativity, Associativity, Idempotency, Identity (element), Absorption (element).

(ii) Using the laws of Propositional Calculus, prove the following:

(a) $((p \wedge q) \wedge r) \wedge s = q \wedge (r \wedge (s \wedge p))$
(b) $\neg(p \vee q) \vee (\neg p \wedge q) = \neg p$
(c) $p \wedge [(p \wedge q) \vee \neg p] = p \wedge q$
(d) $(p \to q) \wedge (q \to p) = (p \wedge q) \vee (\neg p \wedge \neg q)$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

(iii) Prove that Disjunctive Syllogism is a valid inference rule.

B2. (i) Consider the following paradoxes:

P1 S1: The Earth is flat.
S2: Neither of these statements is true (i.e. neither S1 nor S2 is true).

P2 S1: The Moon is made of cheese.
S2: Exactly one of these statements is true (i.e. either S1 is true or S2 is true, but not both).

For each paradox **P1** and **P2**,

(a) can a consistent set of values be assigned to S1 and S2?

(b) if so, what do we deduce about the truth of S1?

(c) explain the source of the paradox in these examples.

(ii) Explain (by giving examples) how (1) the Deduction Theorem and (2) Reductio ad Absurdum can be used to construct proofs in logic.

(iii) Using propositional calculus, determine the validity of the following argument, stating the laws you use:

> *If you place your order by fax or email, then it will be dealt with promptly and efficiently. Your goods will arrive on the next day only if the order is dealt with promptly, or the goods are sent via Priority Post. Therefore, if you place your order by email, your goods will arrive on the next day.*

B3. (i) Given the generalized De Morgan laws from propositional calculus:

$$\neg(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) = \neg p_1 \vee \neg p_2 \vee \neg p_3 \vee \ldots \vee \neg p_n$$

$$\neg(p_1 \vee p_2 \vee p_3 \vee \ldots \vee p_n) = \neg p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge \ldots \wedge \neg p_n$$

prove (for a finite Universe U)

(a) the De Morgan law for Relative Existential Quantification:

$$\neg[\exists x : U \bullet P(x) \wedge Q(x)] = \forall x : U \bullet P(x) \Rightarrow \neg Q(x)$$

(b) the De Morgan law for Relative Universal Quantification:

$$\neg[\forall x : U \bullet P(x) \Rightarrow Q(x)] = \exists x : U \bullet P(x) \wedge \neg Q(x)$$

(ii) Let U be the Universe of people ($x, y \in U$) on which we define the following Atomic Predicates:

$E(x, y)$: x employs y

$C(x)$: x is clever

$T(x)$: x is talkative

Represent the following statements in Predicate Calculus:

(a) Employees are always talkative.

(b) Talkative employees have clever employers.

(c) If all the employers are clever then nobody is talkative.

(d) Clever people are self-employed.

(d) If someone is self-employed then everyone is talkative.