

OLLSCOIL na hÉIREANN, GAILLIMH
 NATIONAL UNIVERSITY of IRELAND, GALWAY

WINTER EXAMINATIONS 2002

SECOND UNIVERSITY EXAMINATION

DISCRETE MATHEMATICS [MA 204]

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Time allowed: **Two** hours.
 Full marks for **three** correct solutions.

1. (a) Suppose 95% of the population like at least one of the three pop stars "The Edge", "The Face" and "The Vertex";
 50% like "The Edge", 36% like "The Face" and 43% like "The Vertex";
 25% like **only** "The Vertex" and 8% like **all three** stars, what percentage of the population likes both "The Edge" and "The Face" but **not** "The Vertex"?
- (b) How many three-digit numbers having **no** repeated digit can be formed using the six digits $\{2,3,4,5,6,7\}$?
 Of these how many are
 - (i) odd?
 - (ii) greater than 400?
 - (iii) odd and greater than 400?
- (c) Wendy and Gail play a draughts match. The first person to win two games in a row or a total of four games wins the match. Assuming no drawn games, construct an appropriate tree diagram to find the number of ways in which the match may have occurred.

2. (a) Show that the number of non-negative integer solutions of

$$x_1 + x_2 + \cdots + x_k = n \quad \text{is} \quad \frac{(n+k-1)!}{n!(k-1)!}.$$

A bookshelf holds 11 different books in a row. By applying the formula above, determine the number of ways we can select 4 books so that no two consecutive books are chosen.

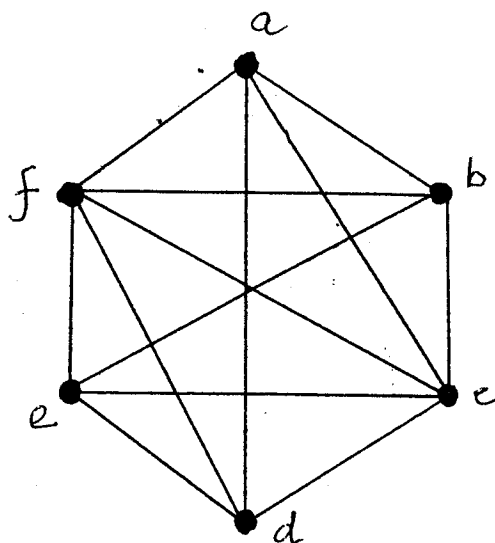
- (b) Find the number of distinct arrangements (permutations) of all the letters in

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- (c) Use a binary search tree to sort the following list in alphabetical order:
matrix, eigenvalue, cofactor, symmetric, diagonal, inverse, elementary, determinant, adjoint, transpose, orthogonal, hermitian

3. (a) Define the term **eulerian graph**. Give a necessary and sufficient condition for a graph to be eulerian.

What is meant by the term **traversable trail**? Show that the graph illustrated below has a traversable trail but is not eulerian.



- (b) Let T be a graph with n vertices and $n - 1$ edges and no cycles. **Prove** that T is a tree.
- (c) Describe an algorithm for finding a spanning tree in a connected graph and use the algorithm to find such a spanning tree in the graph in part (a) above.
- (d) Construct an ordered rooted tree for the algebraic expression

$$\sin^3(\pi t) - 2 \cos(\pi/4)$$

and hence write the expression in Polish notation.

4. (a) For a connected planar map having V vertices, E edges and F faces, **prove** Euler's formula

$$V - E + F = 2.$$

Show that such a map has a vertex of degree at most 5. Determine whether or not the graph illustrated in **Q3 (a)** is planar.

- (b) Show that if a graph G has no vertex of degree greater than k its chromatic number $\chi(G)$ is at most $k + 1$.
- (c) For the graph illustrated in **Q3(a)**, describe the use of the Welch-Powell algorithm to determine its chromatic number.