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OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS 2002

THIRD UNIVERSITY EXAMINATION IN ARTS AND SCIENCE

MATHEMATICS [MA301] ADVANCED CALCULUS

PASS

Dr. Dave Johnson,
Professor T. C Hurley,
Dr. A. Christofides

Time allowed: *Two* hours.

For those in the *Mathematical Studies* group, full marks for **four** questions.
For all other groups full marks for **three** questions.

1. (a) Show, from first principles, that one of the following series converges, while the other diverges. Determine the limit of the one that converges.

$$\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n, \quad \sum_{n=0}^{\infty}\frac{1}{3}.$$

(b) Use the integral test to show that the series

$$S = \sum_{n=1}^{\infty}\frac{1}{n^p}$$

diverges if $p < 1$. Show that the series $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n+1}}{2n+1}$ diverges, either by comparing it with an appropriate series of the form $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^p}$, or otherwise.

2. (a) Use the ratio test to determine the radius of convergence of the following two series

$$(i)\ \sum_{n=1}^{\infty} \frac{x^n}{4^n(n^2+1)}, \quad (ii)\ \sum_{n=1}^{\infty} \frac{x^{2n}}{4^n(n^2+1)}$$

(b) Find the first four terms of the Taylor series of the function $f(x) = 1/\sqrt{x}$ about $x = 1$ and hence determine $1/\sqrt{1.02}$ correct to three decimal places.

3. (a) Let

$$I = \iint_R (2x + 3y)\, dx\, dy,$$

where R is the region contained between the x-axis and the graph of the function $y = 1 - x^2$.

Evaluate I, (i) integrating first with respect to y and then with respect to x, (ii) integrating first with respect to x and then with respect to y, and verify that the answer, in both cases, is the same.

(b) Let V denote the solid bounded by the the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$ whose vertices are the points $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$. Express the volume of V as a double integral and hence evaluate this volume.

4. (a) Let

$$I = \iint_R \sin(x^3 + 1)\, dx\, dy,$$

where R is the region determined by the inequalities $0 \le x \le 1$ and $0 \le y \le x^2$. Explain why the order of integration matters in evaluating the integral I and evaluate I.

(b) Let R be the region in the first quadrant bounded by the x-axis , the y-axis and the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$. Evaluate the integral

$$\iint_R xy\, dx\, dy.$$

p.t.o.

5. (a) Evaluate the line integral

$$\int_C (2x - 3y)\,dx + (x + 2y)\,dy,$$

where C is the line segment from $(2,1)$ to $(4,5)$.

(b) Use Green's theorem to evaluate

$$\int_C (-2xy + x^2)\,dx + (3xy + y^3)\,dy.$$

where C is the triangle with vertices $(0,-2)$, $(1,0)$, $(0,2)$, described in the positive sense.