

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER ONE EXAMINATIONS, 2002–2003

MA 237 – PROBABILITY AND STATISTICS

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Time allowed: **Two Hours.**

Answer the 10 Questions in Section A (20 marks)
and
any 4 Questions from Section B (20 marks each).

Relevant tables and formulæ are supplied

Section A - Compulsory

Each part is worth 2 marks with a loss of 1 mark for a wrong answer.

1. Which measure of central tendency is affected by extreme values in a set of data?
 - (a) Mode
 - (b) Mean
 - (c) Median

2. The following observations are the lifetimes in hours of 10 light bulbs.

818 805 931 886 1136 948 1146 1192 805 821

The median of these is

- (a) 931
 - (b) 908.5
 - (c) 886
3. The set of all possible outcomes of a random experiment is called
 - (a) a random variable
 - (b) an event
 - (c) the sample space

question continued ...

4. If two events A and B are **mutually exclusive** then
- (a) $P(A \cap B) = 0$
 - (b) $P(A \cup B) = 1$
 - (c) $P(A \cap B) = P(A)P(B)$
5. If two events A and B are **independent** then
- (a) $P(A | B) = \frac{P(A)}{P(B)}$
 - (b) $P(A | B) = P(B)$
 - (c) $P(A | B) = P(A)$
6. If 10% of a population are left-handed and we take a random sample of 30 people, then the appropriate probability distribution for the number of left-handers in the sample is a
- (a) Poisson distribution
 - (b) Binomial distribution
 - (c) Geometric distribution
7. Suppose that someone offers you a bet on the roll of a pair of fair dice. If both dice have the same number you win \$30. Otherwise you must pay \$6. Your expected winnings would be
- (a) \$0
 - (b) \$9
 - (c) -\$1
8. For a random variable X with standard deviation 5, $\text{Var}(2X + 4)$ is
- (a) 104
 - (b) 20
 - (c) 100
9. If $Z \sim N(0, 1)$ then $P(0 < Z < 1.645)$ is
- (a) 0.45
 - (b) 0.95
 - (c) 0.475
10. If $X \sim N(3, 4)$ then which of the following has a $N(0, 1)$ distribution?
- (a) $\frac{X - 4}{3}$
 - (b) $\frac{X - 3}{2}$
 - (c) $\frac{X - 3}{4}$

Section B - Answer 4 Questions

1. The following data are wing chord measurements (in centimetres) for 25 female grackles (a type of bird):

12.8	12.0	13.0	12.5	12.8
12.9	12.2	12.8	12.6	12.4
12.3	12.7	12.7	13.5	12.5
12.5	12.8	12.2	12.7	11.2
12.3	12.4	12.8	12.4	12.2

- (a) Construct a stem-and-leaf plot using intervals for the stems of width 0.2. (5)
- (b) Find the five number summary. (5)
- (c) Draw a boxplot. (5)
- (d) Calculate the sample mean and sample standard deviation. (5)
2. (a) How many different ways are there of writing down (in sequence) the letters in the word (3)
- MISSISSIPPI
- How many of these ways include the consecutive string of letters MISS? (2)
- (b) In a batch of 100 light bulbs, there are five defective bulbs. An inspector examines 10 bulbs selected at random. What is the probability of
- i) finding no defectives in the sample? (2)
- ii) finding exactly one defective in the sample? (2)
- iii) finding at least one defective in the sample? (1)
- (c) Suppose that 1% of the population has a particular disease. There is a test on a blood sample which gives one of two results, positive or negative. It is found that 95% of the people having the disease produce a positive result. However, 2% of the people without the disease will also produce a positive result. For a person randomly chosen from the population,
- i) what is the probability that they will test positive? (3)
- ii) given that the person's test was positive, what is the probability that they have the disease? (3)
- (d) The distribution of blood types for a certain population is

40% type A 12% type B 45% type O 3% type AB

If two people are chosen at random from the population, what is the probability that they have the same blood type? (4)

3. (a) The probability that a fluorescent light has a life of over 1000 hours is 0.8. Find the probability that in a random sample of 10 such lights:
- exactly 5 last for more than 1000 hours; (3)
 - at least one does not last for more than 1000 hours; (3)
- If a second sample of 10 such lights is taken, what is the probability that after a thousand hours all of the lights in one of the samples have failed, while 5 of the lights in the other sample are still working? (4)
- (b) The number of calls received at a telephone switchboard is Poisson distributed with mean of 6 calls per hour. Find the probability that
- there are no calls in a given one hour period; (2)
 - there is at least one call in a given one hour period; (2)
 - there are no calls in a given ten-minute period; (3)
 - over a thirty-minute period there are no calls in the first twenty minutes and exactly one call in the remaining ten minutes. (3)

4. The joint probability function of two random variables X and Y is given by the following table:

		Y		
		-1	0	1
X	-1	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{1}{12}$
	0	$\frac{2}{12}$	0	$\frac{2}{12}$
	1	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

- Find the marginal probability functions for X and Y . (3)
- What is the conditional probability that $X = 1$ given that $Y = 0$? (2)
- Calculate the mean and variance of X . (3)
- What can you say about the mean and variance of Y ? (2)
- Calculate the covariance between X and Y . (4)
- Find the correlation between X and Y . (3)
- Are X and Y independent? Give a reason. (3)

5. (a) IQ scores for children are standardised so that they are normally distributed with a mean of 100 and a standard deviation of 15.
- i) What is the probability that a randomly selected child has an IQ less than 80; (2)
 - between 85 and 110? (2)
 - ii) What IQ score is exceeded by only the top 1% of children? (2)
 - iii) In a class of 25 children, what is the probability that the average IQ score of the class is greater than 110? (4)
- (b) An office block has three entrances A , B and C . The numbers of people who enter through these entrances are Poisson distributed with means of $\lambda_A = 3$, $\lambda_B = 5$ and $\lambda_C = 8$ per hour respectively.
- Let N be the total number of people who enter the building per hour. Assuming that the numbers of people who enter through each entrance are independent, what is the distribution of N ? (2)
- Use the normal approximation to the Poisson distribution to calculate the probability
- i) that more than 20 people enter the building in a one hour period; (4)
 - ii) that 150 people or less enter the building during a nine hour period. (4)

Formulæ

- Sample Mean

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

- Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^n x_i^2 - n\bar{x}^2}{n-1}}$$

- Binomial Distribution

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, \dots, n$$

$$E[X] = np \quad \text{Var}[X] = np(1-p)$$

- Poisson Distribution

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad x = 0, 1, \dots$$

$$E[X] = \lambda \quad \text{Var}[X] = \lambda$$

- Variances, Covariances and Correlation

$$\text{Var}[X] = E[X^2] - (E[X])^2$$

$$\text{Cov}[X, Y] = E[XY] - E[X]E[Y]$$

$$\text{Correlation}(X, Y) = \frac{\text{Cov}[X, Y]}{\sqrt{\text{Var}[X] \text{Var}[Y]}}$$