Gx 2079

OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER 1 EXAMINATIONS 2002/2003

MATHEMATICS [MA284] — DISCRETE MATHEMATICS

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Time allowed: *Two* hours.

Answer *three* questions.

1.

(a) A committee of six people is to be chosen from seven married couples. (i) In how many different ways can the committee be chosen? (ii) How many committees contain exactly four women? (iii) How many committees contain at least four women? (iv) How many committees contain no couple? (v) How many committees contain exactly two couples?

(b) Find the number of different arrangements of all the letters in the word

DODECAHEDRON.

How many of these arrangements (i) start with the five vowels; (ii) end with the 3 **D**s; (iii) start with the five vowels and end with the 3 **D**s?

(c) Write out the definition of the binomial coefficient $\binom{n}{k}$ and prove that, for integers $k, n \geq 0$,

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

PTO.

2.

(a) Write out the full PIE formula (Sieve formula) for $n = 4$ subsets A_1, A_2, A_3, A_4 of a given set X. How can the formula be simplified, if intersections of equally many subsets have the same size?

(b) Given a set X of size $|X| = 140$ and subsets A, B, C, and D of X with sizes $|A| = 47$, $|B| = |C| = 55$ and $|D| = 76$. Suppose further that $|A \cap B| = 19$, $|A \cap C| = 15$, $|A \cap D| = 16$, $|B \cap C| = 17$, $|B \cap D| = 25$, $|C \cap D| = 28$, and also that $|A \cap B \cap C| = 5$, $|A \cap B \cap D| = 3$, $|A \cap C \cap D| = 4$, $|B \cap C \cap D| = 2$, and $|A \cap B \cap C \cap D| = 1$. Use the Principle of Inclusion and Exclusion to determine $|X \setminus (A \cup B \cup C \cup D)|$.

(c) There are 500 boxes with apples, each containing no more than x apples. Find the maximal possible value of x, such that one can with certainty find at least 3 boxes containing the same number of apples.

3.

(a) What is a tree? Show that if a graph $G = (V, E)$ is a tree then $|E| = |V| - 1$.

(b) Let G be the graph with vertex set $V = \{a, b, c, d, e, f, g, h, i\}$ and weighted edges as follows: $\{a, b\}$ (weight 5), $\{a, d\}$ (2), $\{b, c\}$ (4), $\{b, d\}$ (3), $\{b, e\}$ (5), $\{b, f\}$ (6), $\{c, f\}$ (3), $\{d, e\}$ (7), $\{d, g\}$ (6), $\{d, h\}$ (8), $\{e, f\}$ (1), $\{e, h\}$ (3), $\{f, h\}$ (4), $\{f, i\}$ (4), $\{g, h\}$ (4), $\{h, i\}$ (2). Draw a diagram for G. Use Kruskal's Algorithm to find a minimum weight spanning tree in G. What is the weight of such a spanning tree?

(c) Construct an ordered rooted tree for the algebraic expression

$$(x + 2)^3(y - (3 + x)) - 5$$

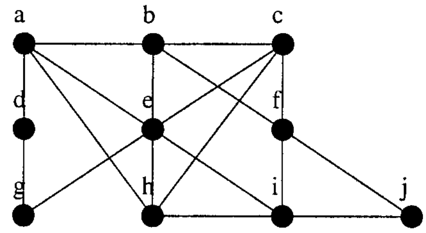
and write this expression in reverse Polish notation. Also, construct an ordered rooted tree for the postfix expression

`9 3 / 5 + 7 2 - *`

and write it as infix expression and determine its value.

4.

(a) Describe briefly the Welsh-Powell algorithm. Use the Welsh-Powell algorithm to determine a colouring of the graph H below.



(b) Define what it means for a graph to be (i) bipartite? (ii) semi-Eulerian? (iii) Eulerian? (iv) Hamiltonian. Is the graph H above (i) bipartite? (ii) semi-Eulerian? (iii) Eulerian? (iv) Hamiltonian? Justify your answers.

(c) (i) What is a planar graph? (ii) State Euler's formula for a connected planar graph. (iii) A planar graph is called a **Platonic graph**, if all vertices have the same degree $\mathsf{a} \geq 3$ and all regions (or faces) have the same degree $\mathsf{b} \geq 3$. Use Euler's formula to show that there are only 5 Platonic graphs.