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OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS 2002-2003

SECOND UNIVERSITY EXAMINATION

MATHEMATICS
MA286 – ANALYSIS
HONOURS

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Time allowed: **Two** hours.
Attempt **three** questions.

1. (a) Sketch the graph of the function defined by

$$f(x, y) = xy.$$

(b) Prove that the following function is not continuous at $(0, 0)$.

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

(c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Show that T is a continuous function.

2. (a) Let $f : \mathbb{R}^2 \to \mathbb{R}$. What does it mean to say that f is differentiable at a point $(x_0, y_0) \in \mathbb{R}^2$? What is the total derivative of f at the point (x_0, y_0)?

(b) Let $f(x, y) = (xy, x^3 y - y^2)$. Compute $T_{(1,2)}f$ (i.e the total derivative of f at the point $(1, 2)$).

(c) Let f be the function defined as follows -

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

i. Compute the directional derivative $f'((1,1);(-11,23))$.

ii. Compute the directional derivative $f'((0,0);(-1,1))$.

3. (a) Find and classify the critical points of the function

$$f(x,y) = x^3 - xy + y^3.$$

(b) Describe *briefly* the method of Lagrange multipliers and use it to solve the following problem. A rectangle is inscribed in the ellipse

$$2x^2 + 5y^2 = 1$$

in such a way that the sides are parallel to the coordinate axes. Find the maximum area that such a rectangle may have.

4. (a) Test the following series for convergence.

i. $\sum_{n=1}^{\infty} \frac{(2n+1)^n}{3^n n^n}$

ii. $\sum_{n=1}^{\infty} \frac{2n^2+2n+1}{n^3-2}$

(b) Let $\sum_{n=0}^{\infty} a_n$ be a series with positive terms such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists. Suppose that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$. Show that the series diverges.

(c) Give an example of a convergent series that is not absolutely convergent and prove that your example has the required properties.

5. (a) Find the interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{n} x^n$$

(b) State Taylor's Theorem. (You are not required to provide a proof of the theorem.)

(c) Let $f(x) = 2\sin x$. Using Taylor's theorem, show that the function f is real analytic.