

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER 1 EXAMINATIONS, 2002-2003

FIRST YEAR FINANCIAL MATHEMATICS AND
SECOND YEAR ARTS

STATISTICS [MA112, MA227]

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Time allowed: *Two* hours.

Answer three questions.

1. (i) The marks obtained by 19 students in a test are

69, 56, 58, 70, 72, 61, 46, 66, 61, 58, 65, 68, 74, 70, 75, 68, 55, 58, 67.

Construct a stem-and-leaf display for the data, and find the median Q_2 and the quartiles Q_1, Q_3 .

- (ii) Find the mean and the standard deviation of the *sample* consisting of the first five marks listed above.
- (iii) It is believed that the mean bed-time for people in Ireland is 11.30 pm, with a standard deviation of 10 minutes. What does Chebyshev's theorem say about the proportion of the population who go to bed between 11.00 pm and midnight?
- (iv) In a national test the mean marks for English, Irish and Mathematics are 69, 62 and 60 respectively, with corresponding standard deviations 4, 5 and 3. Pat's marks are 75, 70 and 65 respectively. Which is Pat's best subject, and why?

p.t.o.

2. (i) There are 3 defective bulbs in a carton of 20, and 2 bulbs are selected at random from the carton. What is the probability that at most 1 of the selected bulbs is defective?
- (ii) State Kolmogorov's axioms for a probability model. Use them to show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

when A and B are any events in a sample space.

- (iii) The events A and B are such that $P(A) = 0.3$ and $P(B) = 0.5$. Find $P(A \cap B)$, $P(A \cup B)$ and $P(A|B)$ when A and B are independent.

3. (i) A factory has 3 machines K, L and M which produce (respectively) 45%, 35% and 20% of its output. Machine K produces 2 faulty items per 100; the figures for L and M are 4 and 5 (per 100). What is the probability that an item from the factory is faulty?
- (ii) State and prove Bayes Theorem.
- (iii) The Rich Sea is divided into blocks for oil exploration, and data suggests there is a 1 in 20 chance of finding oil in any given block. Your company Proboil is licensed to explore a particular block, but before undertaking expensive drilling you use a (cheaper) preliminary test. This test has a false-positive rate of 4% and a false-negative rate of 6%. Given that your block tests positive, what is the probability of finding oil in it?

p.t.o.

4. (i) An unbalanced coin is tossed 64 times. The probability of a head showing on a single toss is $\frac{3}{4}$. Use the normal approximation to the binomial distribution to estimate the probability that a head will occur on exactly 50 of the 64 tosses.
- (ii) The actual amount of beer in a $\frac{1}{2}$ -litre bottle has a normal distribution with a standard deviation of 3 millilitres. What should the mean fill of the bottles be if at most 2% of them are to contain less than a $\frac{1}{2}$ -litre of beer?
- (iii) State the Central Limit Theorem.
The distribution of the heights of Galway students has a mean of 70 ins with a standard deviation of 4 ins. Use the theorem to estimate the probability that the mean height of a sample of 36 students is greater than 71 ins.