GX3088

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER 1 EXAMINATIONS, WINTER 2002

THIRD UNIVERSITY EXAMINATION IN FINANCIAL MATHEMATICS & ECONOMICS

ACTUARIAL MATHEMATICS [MA311] - LIFE CONTINGENCIES

Honours

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Time allowed: *Two* hours
Answer *three* questions.

1. (a) Let $a_{x:\overline{n}|}$ denote the net single premium at age x of an ordinary temporary annuity of €1 per year for n annual payments. Show:

$$a_{x:\overline{n}|} = \frac{N_{x+1} - N_{x+n+1}}{D_x}.$$

(b) Let ${}_yP_{x:\overline{n}|}$ denote the net annual premium for a n-year endowment insurance policy of €1 (if the policy specifies that the net premium is payable in y equal annual amounts) issued at age x. Show:

$${}_yP_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+y}}.$$

(c) A €20,000, 30-year endowment insurance policy was bought by someone aged 35. Find the net annual premium if the policy specifies that the net premium is payable in 15 annual amounts.

We assume 1958 CSO mortality, interest at $2\frac{1}{2}$ % compounded annually, and the death benefit is paid at the end of the year of death.

P.T.O.

2. (a) A policy issued to someone aged 20 promises to pay:

(i) €6,000 if insured dies before reaching 30

(ii) €10,000 if insured dies after reaching 30 but before 55

(iii) €50,000 if insured dies after reaching 55 but before 65, or if the insured is alive at 65.

Find the net annual premium if the policy is a 15-payment policy and if we assume 1958 CSO mortality, interest at $2\frac{1}{2}$ % compounded annually, and the death benefit is paid at the end of the year of death.

(b) Let P be the net annual premium for a policy of €1, t the number of years the policy is in force and V the t^{th} terminal reserve per surviving policyholder for a €1 policy. If the t^{th} terminal reserve is on a date before the date of the last payment of the y annual premiums, show:

$$V = \frac{P(N_x - N_{x+t}) - (M_x - M_{x+t})}{D_{x+t}}.$$

(c) Show

$${}_{s+t}p_x = {}_sp_x \ {}_tp_{x+s} \qquad for \quad s>0,\ t>0, \ \underline{\text{real numbers}}.$$

3. (a) Let T_x be the random future lifetime after age x of a life that survives to age x. Show

$$E[T_x^2] = 2\int_0^{w-x} t \ {}_tp_x \ dt.$$

(b) Let K_x be the curtate future lifetime of a life age x. Show

$$P[K_x = k] = {}_kp_x \, q_{x+k}.$$

(c) Show

$${}_tp_x = e^{-\int_0^t \mu_{x+s} ds}$$

Suppose Gompertz's law is satisfied with $\mu_4 = .016$ and $\mu_5 = .032$. Show

$${}_tp_x = e^{-\frac{.001}{Ln2} 2^x (2^t - 1).}$$

P.T.O.

4. (a) Show

$$\bar{a}_x = \int_0^\infty v^t \, {}_tp_x \, dt$$

(b) Show

$$\ddot{a}_x^{(m)} = \frac{N_x}{D_x} - \frac{(m-1)}{2m}$$

if we assume for fixed $k \in \{0,1,...\}$ that $v^{k+\frac{j}{m}} \, {}_{k+\frac{j}{m}}p_x$ is linear in j for $j = 0,1,..., m-1$.

(c) A certain life table has a one year select period. Suppose

$${}_2p_{[30]} = .94 \text{ and } q_{31} = .02.$$

Find $p_{[30]}$.