

OLLSCOIL NA hÉIREANN, GAILLIMH  
NATIONAL UNIVERSITY OF IRELAND, GALWAY

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SEMESTER 1 EXAMINATIONS, 2002-2003

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FOURTH UNIVERSITY EXAMINATION

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MATHEMATICS [MA495] - LIFE CONTINGENCIES 2

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Dr. D. Johnson  
Professor T. Hurley  
Dr. M. Hayes

Time allowed: *Two* hours.  
Answer **THREE** questions.

In addition to this paper you should have available actuarial tables, logarithmic tables and an electronic calculator which is non-programmable and is not capable of storing text.

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1. (a) A factory has its staff maintained in a stationary condition by employing 45 people each year at exact age 25. If staff retire at age 65 and English Life No. 12 tables - Males mortality is experienced find:
  - (i) the size of the staff
  - (ii) the number of staff that retire each year
  - (iii) the number of pensioners
- (b) If the force of mortality for a single life is given by  $ab^x + cd^x$ ,  $a, b, c, d, > 0$ , find an expression for  $l_x$ , if  $l_0 = 100,000$ .
- (c) A 65 year old man has just retired and has received £40,000 lump sum from his pension scheme. He is considering investing in the following options:
  - (i) The sum payable in twenty half-yearly instalments, the first payable immediately assuming interest at  $2\frac{1}{2}\%$  per annum. Find each instalment.
  - (ii) The annuity, payable quarterly in advance, guaranteed for five years, assuming mortality on the A67/70 table select and 4% interest, with an expense charge of £1 on each payment of the annuity. Find each annuity payment.
  - (iii) The annuity, payable half-yearly in arrears to his wife aged 62 exactly, with proportion to date of death, assuming mortality on the  $a(55)$  female table select and 4% interest with an expense charge of 3% of the purchase money. Find each annuity payment.
- (d) Calculate the profit or loss from mortality for the year to 31st December, 1997, in respect to the business in the table overleaf assuming that all birthdays and renewal dates are 31st December. The deferred annuities are payable yearly from the 60th birthday and sums assured are paid at the end of the year.  
 You may assume 4% interest and mortality experience in line with A67/70 ultimate.

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At 31.12.97				Year 1.1.97 - 31.12.97
Class	Age at entry	No. of years in force	Sum assured or annuity per annum	Contracts ceasing by death: Sum assured or annuity p.a.
Whole-Life Annual Premium	40	10	£100,000	£1,000
Deferred Annuity Single Premium without Return	45	15	£5,000	£100

2. (a) Derive the formula, (defining all symbols used) for the value of an ill-health pension of  $\pounds P$  per year for each year of past and future service for a person aged  $x$ .
- (b) Use the Pension Fund Tables to find the present value of the following pensions for a member aged 40 who joined the pension fund at age 30.
- (i) Pension of  $\pounds 1,000$  per annum for each year of future service, on ill-health retirement.
  - (ii) Pension of  $\pounds 500$  per annum for each year of membership of the fund on retirement due to ill-health.
  - (iii) Pension of  $\pounds 1000$  per annum for each year of future service with an additional pension of  $\pounds 2000$  per annum if retirement does not occur until age 65.
- (c) (i) Contributions to a pension fund by an employee are made at a rate of 5% of salary to age 30, 6% from age 30 to 40 and 7% over age 40. Use the Pension Fund Tables to find the present value of future contributions by a member aged 25 whose salary is  $\pounds 15,000$  per annum.
- (ii) Employee contributions to a pension fund are at the rate of 3% of salary with a deduction from salary of  $\pounds 1000$  per annum to allow for a social welfare payment on retirement. Find an expression for the present value of the future contributions by a member aged  $x$  with salary  $S$ .

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3. (a) Define the dependent central rate of decrement for all causes and for cause  $k$ .
- (b) Find the relationship between the functions in part (a) and the dependent rate of decrement for cause  $k$ ,  $(aq)_x^k$  outlining the assumption you have made.
- (c) Express the independent rate of decrement  $q_x^k$  in terms of the dependent rate of decrement  $(aq)_x^l$ ,  $1 \leq l \leq m$ , where there are  $m$  decrements. State what assumption is used.
- (d) In a multi-decrement table with three decrements, death (d), withdrawal (w) and retirement (r), find the number who die, withdraw and retire from 10,000 persons alive and in service age 60, if

$$(aq)_{60}^w = .08 \quad m_{60}^d = .01 \quad q_{60}^r = .05$$

Verify your answer by showing that the probability of not being removed by any cause over one year at age 60 equals the product of the rates of not being removed for each decrement.

- (e) Given the following combined table where  $b$  are bachelors and  $m$  are married men

Age	Bachelors	Bachelors Dying	Bachelors Marrying	Married Men	Married Men Dying
$x$	$(bl)_x$	$(bd)_x$	$(bh)_x$	$(ml)_x$	$(md)_x$
24	6135	20	541	4372	14
25	5574	19	502	4899	16
26	5053	17	465	5385	18
27	4571	16	430	5832	20

Find:

- (i) The probability that a bachelor aged 25 dies a bachelor within 2 years.
- (ii) The rate of mortality for a married man aged 25.
- (iii) The probability that a bachelor aged 25 dies within two years.

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4. (a) An annual last survivor immediate annuity is payable at the rate of £10,000 per annum while all of the three lives  $x, y$  and  $z$  are alive. At the first death the annuity reduces to £7,000 per annum and at the second death to £5,000 per annum.  
Find an expression for the value of the annuity in terms of single life and joint life annuities.
- (b) Find the single premium at 4% interest for an assurance of £2,000 payable at the end of the year of death of the second life to fail out of three lives aged 30, 40 and 50 given that

$$a_{30:40} = 16.15$$

$$a_{30:50} = 13.93$$

$$a_{40:50} = 13.59$$

$$a_{30:40:50} = 13.08$$

- (c) Use the Z method to express

(i)  $a_{\overline{x:y:\overline{n}|}}$

(ii)  $a_{ab:\overline{xyz}}$

(iii)  $a_{\frac{6}{abcdefg}}$

in terms of certain period annuities and single and joint life annuities.