

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2003

FIRST SCIENCE EXAMINATION

MATHEMATICS
MA 101 — CALCULUS
PASS

First Paper

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Time allowed: *three hours.*
Answer *six* questions only.

1. (a) Evaluate the following limits:

(i) $\lim_{x \rightarrow 3} \frac{3x^2 - 9}{2x - 9}$

(ii) $\lim_{x \rightarrow 1} \frac{x^5 + 2x - 3}{x^4 + x^3 - x - 1}$

(iii) $\lim_{x \rightarrow \infty} \frac{17x^5 + 3x + 2}{8x^5 + 6x^3 + 2x}$

(b) Differentiate the following with respect to x :

(i) $\frac{x \sin x}{x^4 + 1}$

(ii) $(4x^9 + 7x^5 + 3x + 2)^{35}$

(iii) x^{2x+1}

2. Let $h(x) = x^4 - 4x^3 + 20$.

- Find the intervals on which h is increasing, and the intervals on which it is decreasing.
- Find the intervals on which the graph of h is concave up, and the intervals on which it is concave down.
- Find all relative extrema of h and points of inflection of the graph of h .
- Sketch the graph of h , labelling the y -axis intercept and the points found in (c).
Do not use graph paper or a table of values.

p.t.o.

3. (a) Explain (define) the statement " $f(x)$ is continuous at $x = a$ ".
 (b) Suppose

$$f(x) = \begin{cases} kx^3 + 5x + 1 & x \geq 2 \\ 9x^2 + 2 & x < 2. \end{cases}$$

For what value of k is f continuous at $x = 2$?

- (c) Suppose

$$f(x) = \begin{cases} x^3 + 7x + 1 & x \geq 1 \\ 3x^2 + 3x + 3 & x < 1. \end{cases}$$

- (i) Is f continuous at $x = 1$?
 (ii) Find $f'_+(1)$, $f'_-(1)$.

4. (a) State the Mean Value Theorem.
 (b) Prove that if $f'(x) > 0$ for $x \in (a, b)$ then f is increasing on (a, b) .
 (c) Let

$$f(x) = (1 + x)^{\frac{15}{4}} - (1 + 15x/4).$$

- (i) Show that f is increasing on $(0, \infty)$.
 (ii) Deduce from (i) that

$$(1 + x)^{\frac{15}{4}} > 1 + 15x/4$$

for $x > 0$.

5. (a) Find the equation of the tangent line to the graph of $f(x) = \frac{1+x}{1-x}$ at the point on the graph where $x = 0$.
 (b) Sketch the region enclosed by the graphs of $y = \sqrt{x}$ and $y = x$.
 (i) Find the area of the region.
 (ii) Find the volume of the solid generated by revolving this region about the x -axis.

6. Answer *only three* of the following.

- (a) Use the method of substitution to find

$$\int_0^1 3x^2 \sqrt{x^3 + 1} \, dx.$$

- (b) Use partial fractions to find

$$\int \frac{3}{(x-1)(x+2)} \, dx.$$

- (c) Use integration by parts to find

$$\int x \sin x \, dx.$$

- (d) Use the Fundamental Theorem of Calculus to find

$$\frac{d}{dx} \left(\int_0^{x^2} e^{t^2} \, dt \right).$$

p.t.o.

7. (a) Use logarithmic differentiation to find $f'(x)$ if

$$f(x) = \frac{(x^2 + 2)^{3/2}(x^4 + 2x + 1)^3}{(x^3 - 6x)^5}.$$

- (b) Use the partial fractions technique to find

$$\int \frac{5x^2 - 3x + 1}{(x - 2)(x^2 + 1)} dx.$$

8. (a) Solve the differential equation

$$\frac{dy}{dx} + 6x^5y = (\cos x)e^{-x^6}, \quad y(0) = 0.$$

- (b) The population y of a certain village at any time t satisfies the differential equation

$$\frac{dy}{dt} = ky.$$

In 1998 ($t = 0$), the population of the village was 10, and in 2003 ($t = 5$), the population had grown to 100.

- (i) Find k .
- (ii) In which year will the population reach 1000?