

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2003

FIRST EVENING BACHELOR OF COMMERCE
AND
DIPLOMA IN MANAGEMENT EXAMINATION

MATHEMATICS

MA104 - *Quantitative Methods for Business*

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 Ms. D. Quin

Time allowed: *Three* hours
 Answer five questions.

1. (a) Consider the matrices $A = \begin{pmatrix} 4 & 1 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$.

Find:

- (i) A^{-1} , the inverse of A, and
 (ii) B^{-1} , the inverse of B

Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

- (b) Given the following input-output matrix for two industries X and Y:

	Input to X	Input to Y	Final Demand
Output from X	500	280	220
Output from Y	200	560	360

- (i) Complete the above table using the assumption that total input = total output for each sector.
- (ii) Write down the matrix of technical co-efficients.
- (iii) Calculate the total output required from each industry if the final demands from X and Y change to 500 and 1000 units respectively.

2. A company manufactures two types of gates. Each gate must be welded and finished. In addition to this there are also administration hours in the production of each gate. The number of man-hours required to produce each type and the maximum hours available are given in the table below:

Model	Welding	Finishing	Administration
Type 1	6	2	1
Type 2	2	1	1
Maximum Hours	840	300	250

- If x represents the number of gates of type 1 and y represents the number of type 2, write down three inequalities to represent the above constraints.
- Plot these inequalities and shade in the region of feasible solutions.
- Determine using the corner-point method, the number of each type that should be produced to maximize revenue if the selling price per unit is €120 for type 1 and €95 for type 2.
- Calculate, also, the number of man-hours which are not used when revenue is maximized.

3. (a) Let $A = \begin{pmatrix} 2 & 10 & -4 \\ 0 & 3 & 10 \\ 1 & 2 & -2 \end{pmatrix}$

- Find C , the matrix of cofactors of A
 - Calculate A^* , the adjoint of A
 - Calculate $A.A^*$ and hence write down $|A|$, the determinant of A .
 - Use (ii) and (iii) to write down A^{-1} , the inverse of A .
- (b) Consider the system of equations:

$$2x + 10y - 4z = 1$$

$$3y + 10z = 0$$

$$x + 2y - 2z = -1$$

Rewrite this system as: $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

where A is the matrix in (a) and hence solve the system.

4. (a) Differentiate *three* of the following functions with respect to x :

(i) $y = e^{2x}(x^2 - 1)$

(ii) $y = (2 + \sqrt{x})^5$

(iii) $y = \frac{x^2 + 1}{\sin x}$

(iv) $y = (x^3 + \ln x)^7$

- (b) The weekly demand function for a particular product is given by:

$$q = f(p) = 2400 - 15p$$

where q is the quantity demanded and the price p is given in euro.

- (i) Determine the quadratic total revenue function, $R(p)$, where R is a function of p .
- (ii) What is the total revenue at a price of €50 and how many units will be demanded at this price?
- (iii) At what price will total revenue be maximized and what is the total revenue at this price?

5. (a) The total cost of producing q units of a certain product is described by the function

$$C(q) = 5,000,000 + 250q + 0.002q^2,$$

where C is the total cost in euro.

- (i) How many units should be produced in order to minimize the *average cost per unit*?
- (ii) What is the minimum *average cost per unit*?
- (iii) What is the *total cost* of production at this level of output?

- (b) The total cost and revenue functions for a certain product are:

$$C(q) = 40,000 + 25q + 0.002q^2$$

$$R(q) = 75q - 0.008q^2$$

Using the marginal approach, determine the profit-maximizing level of output. What is the maximum profit?

6. (a) The nominal rate of interest on an investment is 6 per cent per annum.
What is the effective annual interest rate (*APR*) if interest is compounded
(i) quarterly (ii) monthly (iii) continuously?
(Answer correct to 2 decimal places)
- (b) Calculate, correct to 2 decimal places, the compound interest rate required for
€10,000 to double in six years if interest is compounded annually.
- (c) A mortgage loan of €100,000 is available at an annual interest 11.25%.
If the period of the loan is 30 years compute:
- (i) the monthly repayments if interest is compounded monthly
 - (ii) the total repayment and
 - (iii) the total interest paid on the mortgage.

Calculate the monthly payments if the mortgage is repaid in 20 years and compute
the difference in interest paid.

7. A sample of results of 122 students yielded the following frequency distribution table:

Score	Frequency
0 – 19	10
20 – 39	12
40 – 59	30
60 – 64	20
65 – 69	22
70 – 79	20
80 – 100	8

- (a) Calculate the mean score of students in the sample.
- (b) Find the standard deviation of their results.
- (c) Display this data using a histogram.
- (d) Estimate the median mark and the lower and upper quartiles.
- (e) Construct a 95% confidence interval for μ , the population mean student mark.
Note: (Answers correct to two decimal places)

8. (a) A commerce student has nine stocks in a shares portfolio of which four are utility stocks and five are industrial stocks. A decision is made to dispose of three of the stocks. If these are chosen at random what is the probability that:
- (i) exactly 2 utility stocks are selected
 - (ii) only utility stocks are selected
 - (iii) at least 1 utility stock is selected?
- (b) In a second-hand car sales lot 50% of cars are Japanese models and of these 15% are hatchbacks; 30% are European and 40% of these are hatchbacks; and finally, 20% are British manufactured and 60% of these are hatchbacks.
- (i) If a car is picked at random from the lot what is the probability that it is a hatchback?
 - (ii) Given that the car chosen is a hatchback, what is the probability that it is a European car?