

Gx2097

OLLSCOIL na hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY of IRELAND, GALWAY

SUMMER EXAMINATIONS, 2003— HONOURS

FIRST ARTS and SCIENCE EXAMINATIONS

FINANCIAL MATHEMATICS and ECONOMICS

MA111 — Mathematics of Finance

Dr Dave Johnson
Professor T.C. Hurley
Dr R.S. Dark
Dr D. O'Regan

Time allowed: *three* hours.
Full marks for *six* questions.

1. (a) €500 was deposited on 22nd November 2002 at 9% simple interest, and the money was withdrawn on 7th February 2003. Find the interest earned on each of the following bases:
 - (i) the exact length of time of the deposit as a proportion of a (non-leap) year;
 - (ii) ordinary simple interest (counting 30 days for each month);
 - (iii) the Bankers' Rule.
- (b) How long will it take for €1,000 to accumulate to €1,200 at 15% simple interest?
- (c) At what simple interest rate will money double in 8 years?
2. (a) Find the amount which must be invested now to receive €1,000 at the end of 3 years, on each of the following bases:
 - (i) using simple discount at 6%;
 - (ii) using compound discount at 6%;
 - (iii) using simple interest at 6%;
 - (iv) using compound interest at 6%.
- (b) For each of the 4 cases in (a) above, find the effective interest rate during the second year.
3. (a) What is the effective interest rate if the nominal rate is 3%, and interest is compounded
 - (i) monthly?
 - (ii) continuously?
- (b) A Credit Union pays an effective interest rate of 7% on deposits at the end of each year. In addition, a bonus of 2% is paid at the end of every 3 years on the balance at that time. Find the effective interest rate earned by an investor if a sum of money is left on deposit for
 - (i) 2 years;
 - (ii) 3 years;
 - (iii) 4 years.
- (c) A retailer has the option of paying a supplier either 30% below the retail price immediately, or 25% below the retail price in 6 months. Find the effective interest rate at which the retailer would be indifferent between the 2 options.

4. Suppose that debts of eS_1 due in t_1 years, eS_2 due in t_2 years and eS_3 due in t_3 years can be settled by a single payment of $eS = S_1 + S_2 + S_3$.

Prove that if i is the effective compound interest rate, and if the single payment is made in t years, then

$$Sv^t = S_1v^{t_1} + S_2v^{t_2} + S_3v^{t_3} \quad \text{where} \quad v = \frac{1}{1+i}.$$

Prove also that if simple interest is used with a comparison date t_0 later than t_1 , t_2 and t_3 , and if the single payment is made in t^* years, then

$$t^* = \frac{S_1t_1 + S_2t_2 + S_3t_3}{S}.$$

Find the values of t and t^* when debts of €600, €300 and €100 are owed in 1 year, 3 years and 6 years respectively, and when the interest rate is 5%.

5. (a) Show that if the effective compound interest rate is $r\%$, then the time t taken for an investment to double is

$$t = \frac{\ln 2}{\ln(1 + r/100)}.$$

Describe the Rule of 72 (which estimates this time) and explain why it works.

- (b) €1,000 is borrowed at 8% effective compound interest. It is to be repaid by 3 equal instalments paid in 2 years, 3 years and 5 years. Find the size of each payment.

6. (a) Let the interest period = payment period and let i be the interest rate per conversion period. Show that the present value of an annuity due of €1 made at the beginning of each period for n equal periods is

$$\frac{1 - (1+i)^{-n}}{i} (1+i).$$

- (b) A house costing €300,000 is purchased by means of a down payment of €100,000 and equal payments at the end of each month for 10 years. The nominal rate of interest is 3% compounded monthly. Find the size of the periodic payments.

- (c) An ordinary annuity, compounded monthly at 4%, pays €200 per month for 2 years, €300 per month for the next year, and €500 per month for the next 2 years. Find the present value of the annuity.

7. (a) Let c be the number of interest periods in one payment period, and i the interest rate per conversion period. Show that the present value of a complex deferred annuity of €1 made at the end of each period for n equal periods, with k deferred payment intervals is

$$a_{\overline{n}|i} \frac{1}{s_{\overline{c}|i}} (1+i)^{-ck}.$$

- (b) The present value of an ordinary annuity of €100 payable at the end of every 6 months for 8 years is €1,500. Use the tables and linear interpolation to find the nominal rate of interest, compounded semi-annually.

8. (a) Let the interest period = payment period, and let i be the interest rate per conversion period. Show that the present value of a simple perpetuity due of €1 is

$$\frac{1+i}{i}.$$

- (b) A leaves an estate of €100,000. Interest on the estate is paid to beneficiary B for the first 8 years, to beneficiary C for the next 12 years, and to charity D thereafter. Find the relative shares of B , C and D in the estate, if it is assumed that the estate will earn a 4% effective rate of return.
9. (a) John's house, painted 5 years ago for €4,000, now needs to be repainted. If John decides to switch to a better paint from now on, he will only need to paint his house every 7 years. How much can he afford to pay for the better paint if the capitalised cost remains the same, given that the effective rate of interest is 5%.
- (b) Let eP be the outstanding loan balance of a debt eA being amortised by equal payments of eR over n periods at rate i per period. Determine P (after k periods) both by the prospective and by the retrospective method, and show the formulas are equal.
10. (a) A loan of €8,000 is being repaid by payments of €1,000 at the end of each month for as long as necessary, plus a smaller final payment. Interest charged is 24% compounded monthly. Find
- the number of payments that will be necessary,
 - the amount of the final payment,
 - the outstanding loan balance after the 4th payment, and
 - the interest paid and the amount the principal is reduced in the 5th payment.
- (b) John borrows €15,000 at a rate of 9% effective. Exactly one year later, John starts accumulating a sinking fund at 10% effective to repay the loan. At the end of each year, John makes a total repayment of €3,000.
- How much of the €3,000 pays the interest in the loan?
 - How much of the €3,000 goes into the sinking fund?
 - What is the balance in the sinking fund at the end of 3 years?
 - How much of the fourth payment should be considered as principal, and how much should be considered as interest?