

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2003

FIRST ARTS EXAMINATION

MA121 - CALCULUS

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Time allowed: *three* hours.

Answer *five* questions.

1. (a) Evaluate the following limits:

$$(i) \lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x^2 - 4} \qquad (ii) \lim_{\theta \rightarrow 0} \frac{\sin 2\theta}{\sin 3\theta}.$$

- (b) Find the horizontal and vertical asymptotes of:

$$f(x) = \frac{3 + 2x - 3x^2}{x^2 - 4x + 3}$$

- (c) Differentiate:

$$\frac{e^{2x} \cos x}{\sin 2x}$$

- (d) Find the equation of the tangent to the curve:

$$3x^3 + ye^x + y^3 = 2$$

at the point $(0, 1)$.

2. Let $f(x) = 3x^3 - 18x^2 + 27x - 10$

- (a) Find the intervals on which f is:
(i) increasing; (ii) decreasing; (iii) concave up; (iv) concave down.
(b) Show that f has a local minimum at $x = 3$, and find the local maximum and the point of inflection.
(c) Find the absolute minimum and absolute maximum of f on the closed interval $[-1, 4]$.

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3. (a) Water is running out of a conical funnel at the rate of 11cm^3 per second. If the radius of the base of the funnel is equal to its vertical height, find the rate at which the water level is dropping when the depth of the water is 14cm .
- (b) An open rectangular tank is to be constructed on a square horizontal base with capacity 2000m^3 . If material for the sides costs 4 euro per square metre and material for the base costs 2 euro per square metre, show that the total cost of material C , is given by:

$$C = 2x^2 + \frac{32,000}{x}$$

where x is the width of the base.

Hence find the dimensions of the cheapest such tank.

4. (a) Evaluate

$$\int_{-1}^4 [x - 2] dx$$

where $[x]$ denotes the greatest integer less than or equal to x (e.g. $[-2.3] = -3$).

- (b) Calculate the area of the region bounded by the x -axis and the curve $y = x^2 + x - 6$.
- (c) A rocket lifts off the surface of the earth with a constant acceleration of 20m/sec^2 . How fast will the rocket be going after 1 minute from lift off? How far will it have travelled in this minute?

5. Calculate three of the following:

(a) $\int \frac{x^2}{(x+2)^3} dx$

(b) $\int x^3 \ln(x) dx$

(c) $\int \sin^{\frac{3}{2}}(x) \cos^3(x) dx$

(d) $\int \frac{1}{(x^2+4)(2x+1)} dx$

6. (a) Define the natural logarithm function $\ln(x)$ as an integral. Define the exponential function $\exp(x)$. Then prove that

$$\ln(xy) = \ln(x) + \ln(y) \quad (x, y > 0),$$

$$\frac{d}{dx} \exp(x) = \exp(x).$$

- (b) The area bounded by the curve $y = x^2 - x$ and line $y = 0$ is revolved about the x -axis. Calculate the volume of the resulting solid.

7. (a) Solve the following differential equation.

$$x \frac{dy}{dx} - 3y = x^4, \quad y(1) = 0.$$

- (b) The population of a certain town was 200 thousand in 1990, and 220 thousand in the year 2000. Predict the population of the town in 2010 assuming that its population y at time t satisfies the differential equation

$$\frac{dy}{dt} = ky$$

for some constant k .